

Safety and Inspection Planning of Older Installations

- **State-of-the art for risk-based inspection methods**
- **Development of new inspection planning methods for older installations**

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Summary

The basic principles in reliability and risk based inspection planning are described. The basic assumption made in risk / reliability based inspection planning is that a Bayesian approach can be used. This implies that probabilities of failure can be updated in a consistent way when new information (from inspections and repairs) becomes available. Further the RBI approach for inspection planning is based on the assumption that in all future inspections no cracks are detected. If a crack is detected then a new inspection plan should be developed. The Bayesian approach and the no-crack detection assumption implies that the inspection time intervals usually become longer and longer.

Further, inspection planning based on the RBI approach implies that single components are considered, one at the time, but with the acceptable reliability level assessed based on the consequence for the whole structure in case of fatigue failure of the component.

Based on the above considerations the following two aspects are considered in this report with the aim to develop the risk based inspection approach, namely

- For aging platform several small cracks are often observed – implying an increased risk for crack initiation (and coalescence of small cracks) and increased crack growth. This should imply shorter inspection time intervals for ageing structures.
- Systems effects including
 - Assessment of the acceptable annual fatigue probability of failure for a particular component taking into account that there can be many fatigue critical components in a structure.
 - Due to common loading, common model uncertainties and correlation between inspection qualities it can be expected that information obtained from inspection of one component can be used not only to update the inspection plan for that component, but also for other nearby components.

Different approaches for updating inspection plans for older installations are proposed. The most promising method consists in increasing the rate of crack initiations at the end of the expected lifetime – corresponding to a bath-tub hazard rate effect. The approach is illustrated for welded steel details in platforms, and implies that inspection time intervals decrease at the end of the platform lifetime.

Data is needed to verify the increased crack initiation model. These data can be direct observations of cracks in older installations or indirect information from inspection programmes.

The different principal system effects are described, and a possible implantation in the generic inspection framework is described.

The approaches described is especially developed for inspection planning of fatigue cracks, but can also be used for various other deterioration processes where inspection is relevant, including corrosion, chloride ingress in concrete and possible corrosion of reinforcement and wear.

Table of Content

1	Introduction	3
2	Risk Based Inspection Planning – state-of-the-art	5
2.1	Acceptance criteria for individual joints	5
2.2	Optimal reliability-based inspection planning	8
2.3	Risk-based inspection planning	12
2.4	Probabilistic modeling of inspections	13
2.5	Probabilistic Fatigue Modelling	13
2.5.1	Assessment of SN Fatigue Lives	13
2.5.2	Assessment of FM Fatigue Lives	15
2.6	Implementation of Generic Inspection Planning	18
2.6.1	iPlan	18
2.6.2	Inspection Planning of Jackets	19
2.7	Examples	20
2.7.1	Example 1.1	20
2.7.2	Example 1.2	21
2.7.3	Example 2	22
3	Inspection planning and systems effects for older installations – Platforms.....	25
4	Inspection planning for older - modified models for stochastic parameters	27
4.1	Modified stochastic models for older structures	28
4.2	Examples	30
5	Systems effects	39
5.1	Aspect a – acceptable annual fatigue probability of failure	39
5.2	Aspect b – update inspection plan based on inspection of other components	41
5.3	Aspect c – effect of redistribution of load effects due to growing cracks	45
6	Summary.....	46
7	References	47

1 Introduction

Reliability and risk based inspection planning (RBI) for offshore structures have been an area of high practical interest over the last three decades. The first developments were within inspection planning for welded connections subject to fatigue crack growth in fixed steel offshore platforms. This application area for RBI is now the most developed. In the beginning practical application of RBI required a significant expertise in the areas of structural reliability theory and fatigue and fracture mechanics, see e.g. PIA [2]. This made practical implementation in industry difficult. Recently generic and simplified approaches for RBI have been formulated making it possible to base inspection planning on a few key parameters commonly applied in deterministic design of structures, e.g. the Fatigue Design Factor (FDF) and the Reserve Strength Ratio (RSR), see Faber et al. [14, 16].

Based on the results of detailed sensitivity studies with respect to the “generic parameters” such as the bending to membrane stress ratio, the design fatigue life and the material thickness, a significant number of inspection plans are computed by a simulation technique for fixed generic parameters (pre-defined generic plans). These generic plans are collected in a database and used in such a way that inspection plans for a particular application can be obtained by interpolation between the pre-defined generic plans. The database facilitates the straightforward production of large numbers of inspection plans for structural details subject to fatigue deterioration. The above state-of-the-art is described in section 2. The use of the generic approach is illustrated on an example.

The basic assumption made in risk / reliability based inspection planning is that a Bayesian approach can be used. This implies that probabilities of failure can be updated in a consistent way when new information (from inspections) becomes available. Further the RBI approach for inspection planning is based on the assumption that at all future inspections no cracks are detected. If a crack is detected then a new inspection plan should be developed. The Bayesian approach and the no-crack detection assumption implies that the inspection time intervals usually become longer and longer.

Further, inspection planning based on the RBI approach implies that single components are considered, one at the time, but with the acceptable reliability level assessed based on the consequence for the whole structure in case of fatigue failure of the component.

Examples and information on reliability-based inspection and maintenance planning can be found in a number of papers, e.g. Thoft-Christensen P. & Sørensen [1], Madsen, Sørensen & Olesen [2], Madsen & Sørensen [3], Fujita, Schall & Rackwitz [4], Skjong [5], Sørensen, Faber, Rackwitz & Thoft-Christensen [6], Faber & Sørensen [7], Ersdal [8], Sørensen, Straub & Faber [9], Moan [10], Kübler & Faber [11], Straub & Faber [12], Rouhan & Schoefs [13], Faber, Sørensen Tychsen & Straub [14], PIA [15] and Faber, Englund, Sorensen & Bloch [16]. Important aspects are systems considerations, design using robustness considerations by accidental collapse limit states and use of monitoring by the leak before break principle to identify damage

Based on the above considerations the following two aspects are considered in this report with the aim to develop the risk based inspection approach, namely

- For aging platform several small cracks are often observed – implying an increased risk for crack initiation (and coalescence of small cracks) and increased growth – thus modelling a bath-tub effect. This should imply shorter inspection time intervals for ageing structures.

- Systems effects including
 - Assessment of the acceptable annual fatigue probability of failure for a particular component taking into account that there can be a number of fatigue critical components in a structure.
 - Due to common loading, common model uncertainties and correlation between inspection qualities it can be expected that information obtained from inspection of one component can be used not only to update the inspection plan for that component, but also for other nearby components.

Initiation of several small cracks implies that these can coalesce to larger cracks which can grow and become critical. The many small cracks also implies that larger cracks can initiate at more than one position, i.e. a systems effect along the welding can be of importance depending on the length of the weld and the dependence between the fatigue cracks.

The inspection updating approach for older platforms is considered in section 4.

The different principal system effects are described in section 5, and a possible implan-tation in the generic inspection framework is described.

2 Risk Based Inspection Planning – state-of-the-art

This section describes and illustrate by examples the basic ideas in risk based inspection planning based on references [14-16].

2.1 Acceptance criteria for individual joints

Requirements to the safety of offshore structures are commonly given in two ways. In the North Sea it is a requirement that the offshore operator demonstrates to the authorities that risk to personnel and risk to the environment are controlled and maintained within acceptable limits throughout the operational service life of the installation. The limits are usually determined in agreement between the authorities and the offshore operator.

Normally, the requirements to the acceptable risk are given in terms of an acceptable Fatal Accident Rate (FAR) for the risk of personnel and in terms of acceptable frequencies of leaks and outlets of different categories for the risk to the environment. These acceptance criteria address in particular risk associated with the operation of the facilities on the topside and cannot be applied directly as a basis for the inspection planning of the structural components.

In addition to the general requirements stated above also indirect and direct specific requirements to the safety of structures and structural components are given in the codes of practice for the design of structures. As an example the NKB [17] specifies a maximum annual probability of failure of 10^{-5} for structures with severe consequences of failure. For offshore structures no codes as of yet give specific requirements to the acceptable failure probability.

In regard to fatigue failures the requirements to safety are typically given in terms of a required Fatigue Design Factor (FDF). As an example NORSOK [17] specifies the FDF's specified in Table 1.

Classification of structural components based on damage consequence	Access for inspection and repair		
	No access or in the splash zone	Accessible	
		Below splash zone	Above splash zone
Substantial consequences	10	3	2
Without substantial consequences	3	2	1

Table 1. Fatigue Design Factors. Factors relate to 'mean \div 2 standard deviation' SN-curves.

"Substantial consequences" in this context means that failure of the joint will entail:

- Danger of loss of human life;
- Significant pollution;
- Major financial consequences.

By "Without substantial consequences" is understood failure, where it can be demonstrated that the structure satisfy the requirement to damaged condition according to the Accidental Limit States with failure in the actual joint as the defined damage.

From the FDF's specified in Table 1 it is possible to establish the corresponding annual probabilities of failure for a specific year. In principle the relationship between the FDF and the annual probability of failure has the form shown in Figure 1.

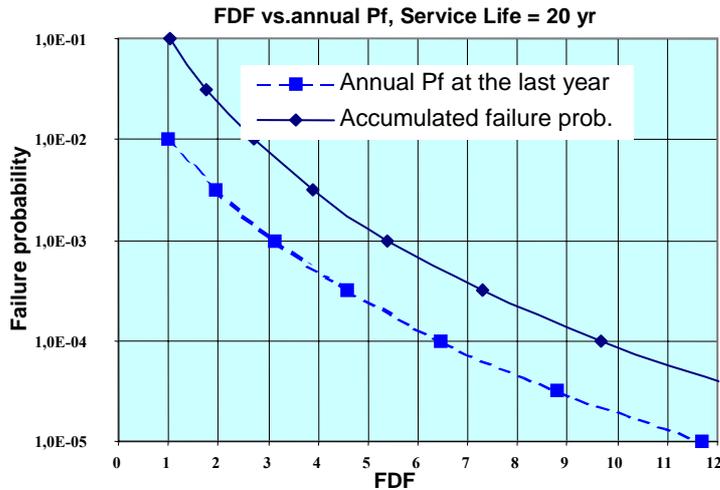


Figure 1. Example relationship between FDF and probability of fatigue failure.

For the joints to be considered in an inspection plan, the acceptance criteria for the annual probability of fatigue failure may be assessed through the RSR given failure of each of the individual joints to be considered together with the annual probability of joint fatigue failure.

If the RSR given joint fatigue failure is known (can be obtained from e.g. an USFOS analysis), it is possible to establish the corresponding annual collapse failure probability given fatigue failure, $P_{COL|FAT}$ if information is available on

- applied characteristic values for the capacities
- applied characteristic values for the live loads
- applied characteristic values for the wave height, period, ... (environmental load)
- ratios of the environmental load to the total load
- coefficient of variation of the capacity and the load

In order to assess the acceptable annual probability of fatigue failure for a particular joint in a platform the reliability of the considered platform must be calculated conditional on fatigue failure of the considered joint. The importance of a fatigue failure is measured by the Residual Influence Factor defined as

$$RIF = \frac{RSR^{\text{damaged}}}{RSR^{\text{intact}}} \tag{1}$$

where RSR^{intact} is the RSR value for the intact structure and RSR^{damaged} is the RSR value for the structure damaged by fatigue failure of a joint.

The principal relation between RIF and annual collapse probability is illustrated in Figure 2.

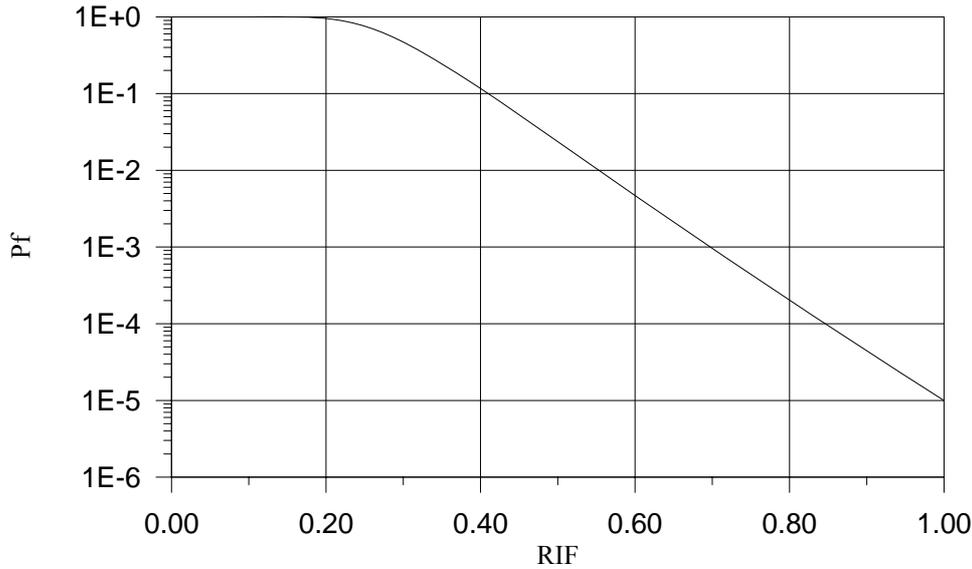


Figure 2. Example relationship between Residual Influence Factors (RIF) and annual collapse probability of failure.

The implicit code requirement to the safety of the structure in regard to total collapse may be assessed through the annual probability of joint fatigue failure (in the last year in service) P_{FAT_j} for a joint for which the consequences of failure are “substantial” (i.e. design fatigue factor 10). This probability can be regarded an acceptance criteria i.e. P_{AC} . A typical maximal allowed annual probability of collapse failure is in the order of 10^{-5} .

On this basis it is possible to establish joint & member specific acceptance criteria in regard to fatigue failure. For each joint j the conditional probabilities of structural collapse give failure of the considered joint $P_{COL|FAT_j}$ are determined and the individual joint acceptance criteria for the annual probability of joint fatigue failure are found as

$$P_{AC_j} = \frac{P_{AC}}{P_{COL|FAT_j}} \quad (2)$$

The inspection plans must then satisfy that

$$P_{FAT_j} \leq P_{AC_j} \quad (3)$$

for all years during the operational life of the structure.

The annual probability of joint fatigue failure P_{FAT_j} may in principle be determined on the basis of either a simplified probabilistic SN approach or a probabilistic fracture mechanics approach provided the fracture mechanical model has been calibrated to the appropriate SN model.

As an alternative to the above approach where basis is taken in annual probabilities of failure it is equally possible to take basis in service life probabilities. However, as most installation concept risk analysis give requirements to the maximum allowable risk for

structural collapse in terms of annual failure probabilities, these are used in the following.

In addition to the acceptance criteria relating to the maximum allowable annual probabilities of joint fatigue failure, economical considerations can be applied as basis for the inspection planning. The aim is to plan inspections such that the overall service life costs are minimised. The costs include costs of failure, inspections, repairs and production losses, see next section.

Ersdal [8] considered life extension of existing offshore jacket structures including fatigue degradation and inspection effects in a life extension. A predictive Bayesian approach is used. Different inspection and repair methods are considered indicating that degradation of the structure due to fatigue crack growth can be controlled by inspections and repair for a significant extended life. Investigations show that systems effects related to life extension and possible combined hazard of wave-in-deck loading are found to be very important.

2.2 Optimal reliability-based inspection planning

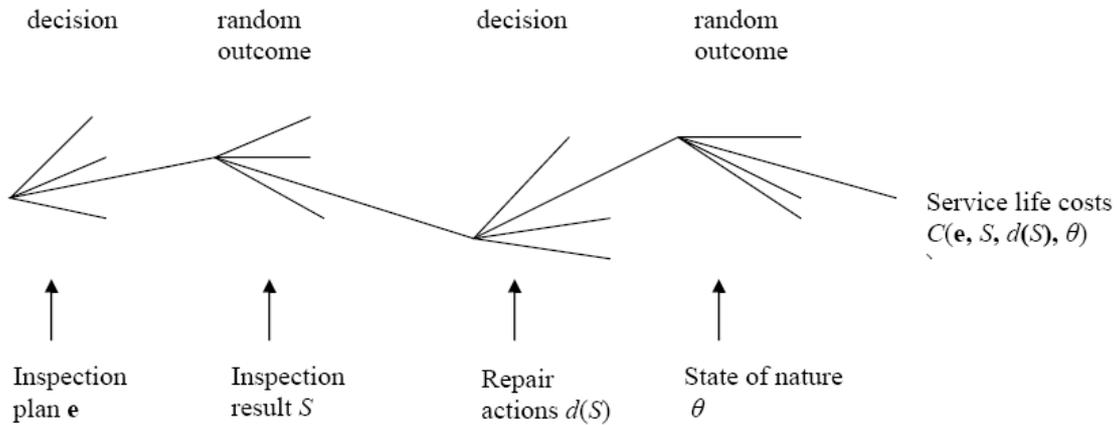


Figure 3. Inspection planning decision tree.

The decision problem of identifying the cost optimal inspection plan may be solved within the framework of pre-posterior analysis from the classical Bayesian decision theory see e.g. Raiffa and Schlaifer [19] and Benjamin and Cornell [20]. Here a short summary is given following Sørensen et al. [6]. The inspection decision problem may be represented as shown in Figure 3.

In the general case the parameters defining the inspection plan are

- the possible repair actions i.e. the repair decision rule d
- the number of inspections N in the service life T_L
- the time intervals between inspections $\mathbf{t} = (t_1, t_2, \dots, t_N)$
- the inspection qualities $\mathbf{q} = (q_1, q_2, \dots, q_N)$.

These inspection parameters are written as $\mathbf{e} = (N, \mathbf{t}, \mathbf{q})$. The outcome, typically a measured crack size, of an inspection is modelled by a random variable S . A decision rule d is then applied to the outcome of the inspection to decide whether or not repair should be performed. The different uncertain parameters (stochastic variables) modelling the

state of nature such as load variables and material characteristics are collected in a vector $\mathbf{X} = (X_1, X_2, \dots, X_n)$.

If the total expected costs are divided into inspection, repair, strengthening and failure costs and a constraint related to a maximum yearly (or accumulated) failure probability ΔP_F^{\max} related to P_{AC_j} for joint j is added, then the optimisation problem can be written

$$\begin{aligned} \min_{\mathbf{e}, d} \quad & C_T(\mathbf{e}, d) = C_{IN}(\mathbf{e}, d) + C_{REP}(\mathbf{e}, d) + C_F(\mathbf{e}, d) \\ \text{s.t.} \quad & \Delta P_{F,t} \leq \Delta P_F^{\max} \quad t = 1, 2, \dots, T_L \end{aligned} \quad (4)$$

$C_T(\mathbf{e}, d)$ is the total expected cost in the service life T_L , C_{IN} is the expected inspection cost, C_{REP} is the expected cost of repair and C_F is the expected failure cost. The annual probability of failure in year t is $\Delta P_{F,t}$. The N inspections are assumed performed at times $0 \leq T_1 \leq T_2 \leq \dots \leq T_N \leq T_L$.

If the repair actions are 1) to do nothing, 2) to repair by welding for large cracks, and 3) to repair by grinding by small cracks, then the number of branches becomes 3^N . It is noted that generally the total number of branches can be different from 3^N if the possibility of individual inspection times for each branch is taken into account.

The total capitalised expected inspection costs are

$$C_{IN}(\mathbf{e}, d) = \sum_{i=1}^N C_{IN,i}(\mathbf{q})(1 - P_F(T_i)) \frac{1}{(1+r)^{T_i}} \quad (5)$$

The i th term represents the capitalized inspection costs at the i th inspection when failure has not occurred earlier, $C_{IN,i}(q_i)$ is the inspection cost of the i th inspection, $P_F(T_i)$ is the probability of failure in the time interval $[0, T_i]$ and r is the real rate of interest.

The total capitalised expected repair costs are

$$C_{REP}(\mathbf{e}, d) = \sum_{i=1}^N C_{R,i} P_{R_i} \frac{1}{(1+r)^{T_i}} \quad (6)$$

$C_{R,i}$ is the cost of a repair at the i th inspection and P_{R_i} is the probability of performing a repair after the i th inspection when failure has not occurred earlier and no earlier repair has been performed.

The total capitalised expected costs due to failure are estimated from

$$C_F(\mathbf{e}, d) = \sum_{t=1}^{T_L} C_F(t) \Delta P_{F,t} P_{COL|FAT_j}(RSR) \frac{1}{(1+r)^t} \quad (7)$$

where $C_F(t)$ is the cost of failure at the time t and $P_{COL|FAT_j}(RSR)$ is the conditional probability of collapse of the structure given fatigue failure of the considered component j .

Details on the formulation of limit state equations for the modelling of failure, detection and repair events are given in Sørensen et al. [6]. Finally, the cumulative probability of failure at time T_i , $P_F(T_i)$ may be found by summation of the annual failure probabilities

$$P_F(T_i) = \sum_{t=1}^T \Delta P_t P_{COL|FAT_j} \quad (8)$$

The solution of the optimization problem (4) in its general form is difficult to obtain. However, if as an approximation it is assumed that all the components of $\mathbf{P}_f^T = (P_{f1}^T, P_{f2}^T, \dots, P_{fL}^T)^T$ are identical ($=P_f^T$), i.e. that the same threshold on the annual probability of failure is applied for all years, the problem is greatly simplified. In this case (4) may be solved in a practical manner by performing the optimization over P_f^T outside the optimization over d and e . The total expected cost corresponding to an inspection plan evolving from a particular value of P_f^T is then evaluated over a range of values of P_f^T and the optimal $P_f^* = P_f^T$ is identified as the one yielding the lowest total costs.

In order to identify the inspection times corresponding to a particular P_f^T another approximation is introduced, namely that all the future inspections will result in no-detection. Thereby the inspection times are identified as the times where the annual conditional probability of fatigue failure (conditional on no-detection at previous inspections) equals P_f^T . This is clearly a reasonable approximation for components with a high reliability, see Straub [21].

Having identified the inspection times the expected costs are evaluated. It is important to note that the probabilities entering the cost evaluation are not conditioned on the assumed no-detection at the inspection times. This in order to include all possible contributions to the failure and repair costs.

The process is repeated for a range of different values of P_f^T and the value P_f^* , which minimizes the costs and at the same time fulfils the given requirements to the maximum acceptable P_f^T is selected as the optimal one. The optimal inspection plan is then the inspection times $0 \leq T_1 \leq T_2 \leq \dots \leq T_N \leq T_L$ corresponding to P_f^* , the related optimal repair decision rule d together with the inspection qualities q .

Following the approach outlined above it is possible to establish so-called generic inspection plans. The idea is to pre-fabricate inspection plans for different joint types designed for different fatigue lives. For given

- Type of fatigue sensitive detail – and thereby code-based SN-curve
- Fatigue strength measured by *FDF* (Fatigue Design Factor)
- Importance of the considered detail for the ultimate capacity of the structure, measured by e.g. *RIF* (Residual Influence Factor)
- Member geometry (thickness)
- Inspection, repair and failure costs

the optimal inspection plan i.e. the inspection times, the inspection qualities and the repair criteria, can be determined. This inspection plan is generic in the sense that it is representative for the given characteristics of the considered detail, i.e. SN-curve, *FDF*, *RSR* and the inspection, repair and failure costs.

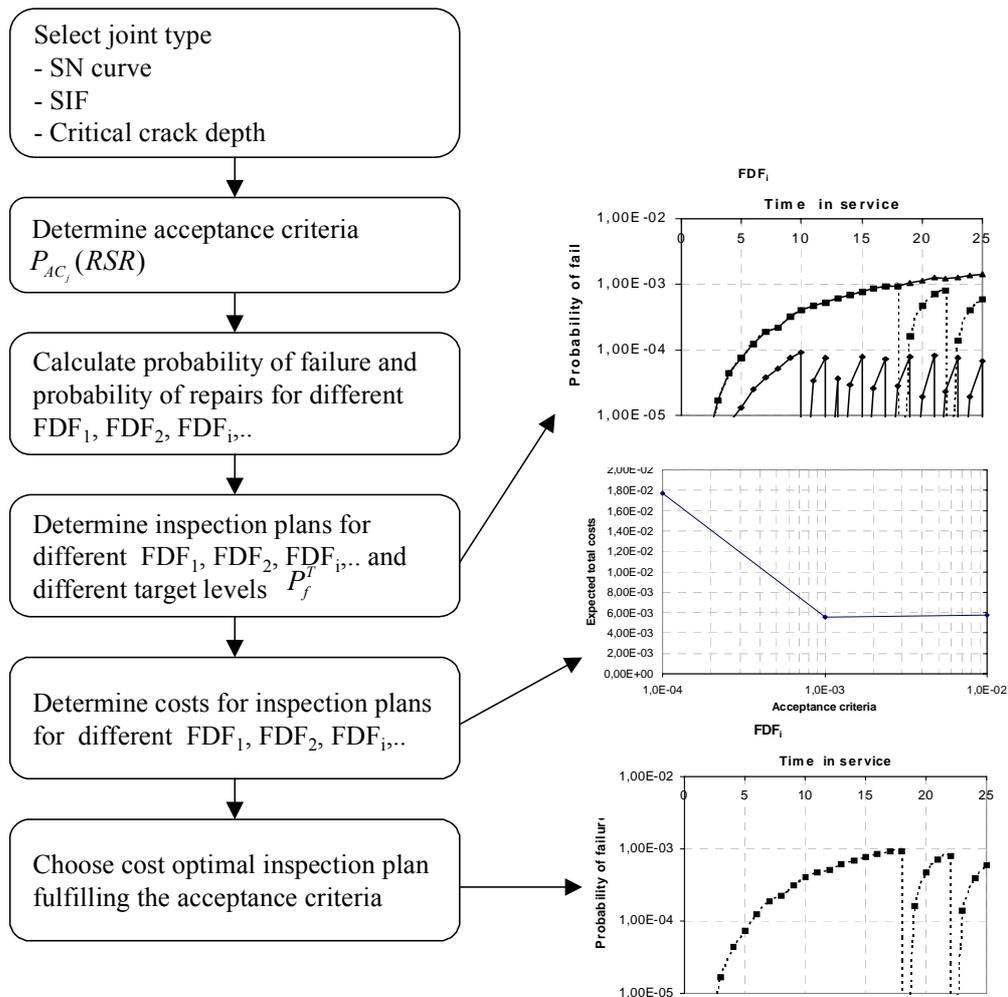


Figure 4. Illustration of the flow of the generic inspection planning approach.

For given SN-curve, member geometry, FDF and cost structure the procedure may be summarized as follows:

1. Identify inspection times by assuming inspections at times when the annual failure probability exceed a certain threshold.
2. Calculate the probabilities of repairs corresponding to the times of inspections
3. Calculate the total expected costs.
4. Repeat steps 1-3 for a range of different threshold values and identify the optimal threshold value as the one yielding the minimum total costs.

The inspection times corresponding to the optimal threshold value then represent the optimal inspection plan. For the identification of optimal inspection methods and repair strategies the above mentioned procedure may be looped over different choices of these. The procedure is illustrated in Figure 4.

As the generic inspection plans are calculated for different values of the FDF it is possible to directly assess the effect of design changes or the effect of strengthening of joints on existing structures as such changes are directly represented in changes of the FDF . It is furthermore interesting to observe that the effect of service life extensions on the required inspection efforts may be directly assessed through the corresponding

change on the *FDF*. Given the required service life extension, the *FDF* for the joint is recalculated and the corresponding pre-fabricated inspection plan identified.

2.3 Risk-based inspection planning

The inspection planning procedure described in the above section requires information on costs of failure, inspections and repairs. Often these are not available, and the inspection planning is based on the requirement that the annual probability of failure in all years has to satisfy the reliability constraint in (4). This imply that the annual probabilities of fatigue failure has to fulfill (3). Further, in risk-based inspection planning the planning is often made with the assumption that no cracks are found at the inspections. If a crack is found, then a new inspection plan has to be made based on the observation.

If all inspections are made with the same time intervals, then the annual probability of fatigue failure could be as illustrated in figure 5.

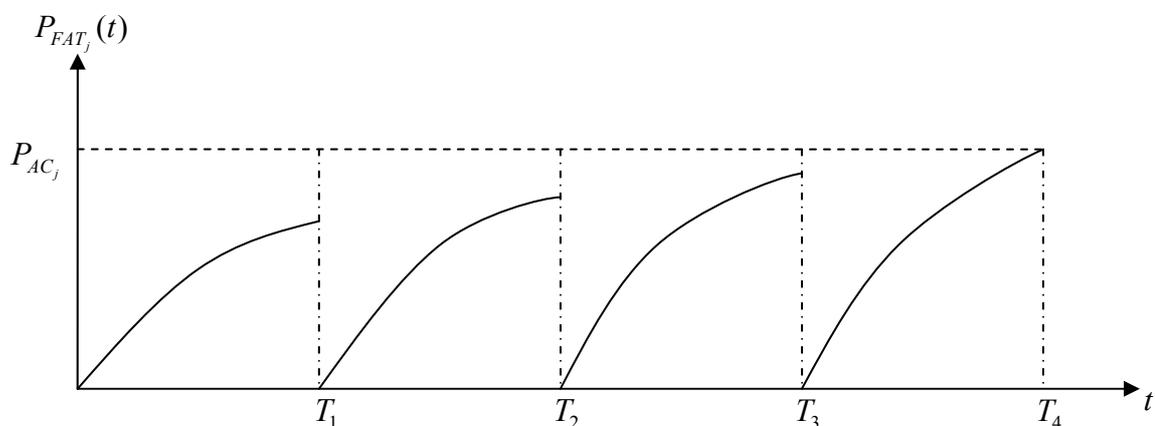


Figure 5. Illustration of inspection plan with equidistant inspections.

If inspections are made when the annual probability of fatigue failure exceeds the critical value then inspections are made with different time intervals, as illustrated in figure 6. The inspection planning is based on the no-find assumption. This way of inspection planning is the one which is most often used. Often this approach results in increasing time intervals between inspections.

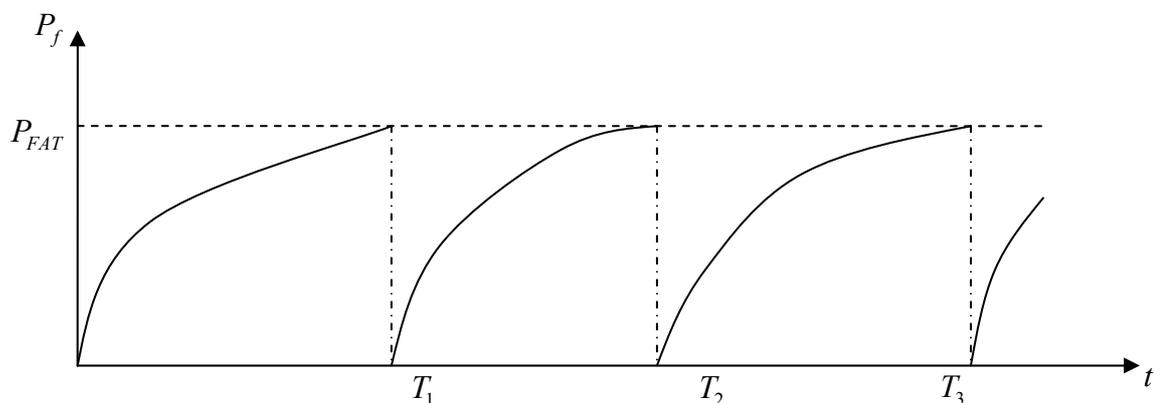


Figure 6. Illustration of inspection plan where inspections are performed when the annual probability of failure exceeds the maximum acceptable annual probability of failure.

2.4 Probabilistic modeling of inspections

The reliability of inspections can be modelled in many different ways. Often POD (Probability Of Detection) curves are used to model the reliability of the inspections.

If inspections are performed using an Eddy Current technique (below or above water) or a MPI technique (below water) the inspection reliability can be represented by following Probability Of Detection (POD) curve:

$$POD(x) = 1 - \frac{1}{1 + \left(\frac{x}{x_0}\right)^b} \quad (9)$$

where e.g. $x_0 = 12.28$ mm and $b = 1.785$.

Other models such as exponential, lognormal and logistics models can be used.

The measurement uncertainty may be modelled by a Normal distributed random variable ε with zero mean value and standard deviation $\sigma_\varepsilon = 0.5$ mm.

Also the Probability of False Indication (PFI) can be introduced and modelled probabilistically.

2.5 Probabilistic Fatigue Modelling

In this section the probabilistic models for fatigue assessment based on SN-curves and fracture mechanics are briefly summarized.

2.5.1 Assessment of SN Fatigue Lives

If a bilinear SN-curve is applied the SN relation can be written:

$$N = K_1 \left(\frac{\Delta s}{(T/T_{ref})^{\alpha^*}} \right)^{-m_1} \quad \text{for } N \leq N_C \quad (10)$$

$$N = K_2 \left(\frac{\Delta s}{(T/T_{ref})^{\alpha^*}} \right)^{-m_2} \quad \text{for } N > N_C \quad (11)$$

where Δs : stress range, N : number of cycles to failure, K_1, m_1 : material parameters for $N \leq N_C$, K_2, m_2 : material parameters for $N > N_C$, Δs_C : stress range corresponding to N_C , T : thickness, T_{ref} : reference thickness and α^* : scale exponent.

Further it is assumed that the total number of stress ranges for a given fatigue critical detail can be grouped in n_σ groups / intervals such that the number of stress ranges in group i is n_i per year. The code-based design equation is then written:

$$G = 1 - \sum_{s_i \geq \Delta s_C} \frac{n_i T_F}{K_1^C S_i^{-m_1}} - \sum_{s_i < \Delta s_C} \frac{n_i T_F}{K_2^C S_i^{-m_2}} = 0 \quad (12)$$

where

$$s_i = \frac{Q_i}{z} \frac{1}{(T/T_{ref})^{\alpha^*}} = \frac{Q_i}{z^*} \quad \text{stress range in group } i$$

Q_i action effect (proportional to stress range s_i in group i)

z design parameter

z^* modified design parameter taking into account thickness effects

K_i^C characteristic value of K_i (mean of $\log K_i$ minus two standard deviations of $\log K_i$)

T_F fatigue life

The design parameter z^* is determined from the design Equation (12). Next, the reliability index (or the probability of failure) is calculated using this design value and the limit state function associated with (12). The limit state equation can be written:

$$g = \Delta - \sum_{s_i \geq \Delta s_c} \frac{n_i T_L}{K_1 s_i^{-m_1}} - \sum_{s_i < \Delta s_c} \frac{n_i T_L}{K_2 s_i^{-m_2}} \quad (13)$$

where

Δ model uncertainty related to Palmgren-Miners rule for linear damage accumulation

$$s_i = X_S \frac{Q_i}{z^*} \quad \text{stress range in group } i$$

X_S stochastic variable modeling model uncertainty related to waves and SCF (wave load response). X_S is Log-Normal distributed with mean value = 1 and $COV = \sqrt{COV_{wave}^2 + COV_{SCF}^2}$. The coefficient of variation COV_{wave} models the uncertainty on the wave load, foundation stiffness and stress ranges. COV_{SCF} models the uncertainty in the stress concentration factors (SCF) and local joint flexibilities (LFJ).

K_i $\log K_i$ is modeled by a Normal distributed stochastic variable according to a specific SN-curve. Two SN-curves (T and F) are used as illustration in the following.

T_L service life

Using the stochastic model in Table 2 and Equation (13) the probability of failure in the service life and the annual probability of failure is obtained.

An alternative stochastic model is to model the long term distribution of fatigue stress ranges by a Weibull distribution, where the parameters itself are uncertain modeling the uncertainty related to the wave load and stress determination.

It is noted that the uncertainties related to Δ and K_i should be modeled carefully. The uncertainty related to Δ (variable amplitude loading and linear damage accumulation by Miner's rule) can be significant. However, in many cases this uncertainty is included in the stochastic model for K_i , e.g. for welded tubular joints.

Variable	Distribution	Expected value	Standard deviation
Δ	LN	1	0 / 0.3
Z_{SCF}	LN	1	$COV_{SCF} = 0.00 / 0.05 / 0.10$ (F-curve) $COV_{SCF} = 0.15 / 0.20$ (T-curve)
Z_{wave}	LN	1	$COV_{wave} = 0.10 / 0.15 / 0.30$
T_F	D	25 – 400 years	
T_L	D	T_F / FDF	
m_1	D	3	
$\log K_1$	N	12.048 (F) 12.713 (T)	0.218 0.200
m_2	D	4	
$\log K_2$	N	13.980 (F) 14.867 (T)	0.291 0.267
log K_1 and log K_2 are assumed fully correlated			

Table 2. Example of stochastic model. D: Deterministic, N: Normal, LN: LogNormal.

2.5.2 Assessment of FM Fatigue Lives

A fracture mechanical modeling of the crack growth is applied assuming that the crack can be modeled by a 2-dimensional semi-elliptical crack. It is assumed that the fatigue life may be represented by a fatigue initiation life and a fatigue propagation life. It is therefore:

$$N = N_I + N_P \quad (14)$$

where

N number of stress cycles to failure

N_I number of stress cycles to crack propagation

N_P number of stress cycles from initiation to crack through.

The number of stress cycles from initiation to crack through is determined on the basis of a two-dimensional crack growth model. The crack is assumed to be semi-elliptical with length $2c$ and depth a , see Figure 7.

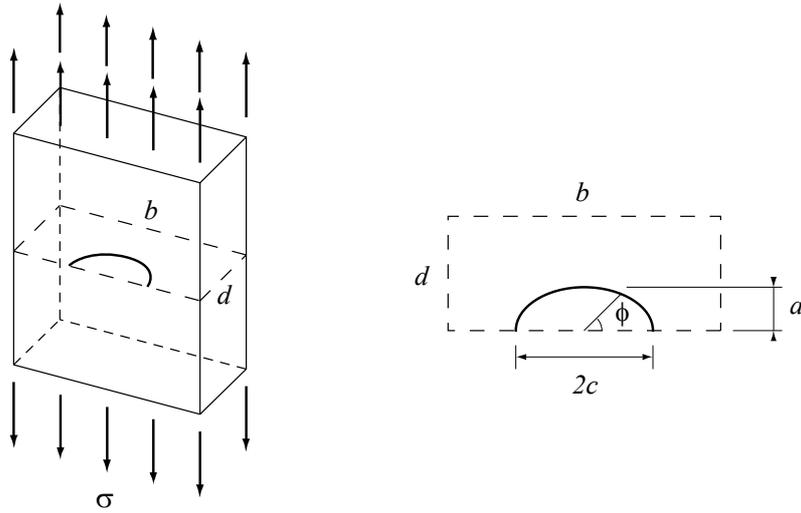


Figure 7. Semi-elliptical surface crack in a plate under tension or bending fatigue loads.

The crack growth can be described by the following two coupled differential equations.

$$\begin{aligned} \frac{da}{dN} &= C_A (\Delta K_A)^m & a(N_0) &= a_0 \\ \frac{dc}{dN} &= C_C (\Delta K_C)^m & c(N_I) &= c_0 \end{aligned} \quad (15)$$

where C_A , C_C and m are material parameters, a_0 and c_0 describe the crack depth a and crack length c , respectively, after N_I cycles and where the stress intensity ranges are ΔK_A and ΔK_C . ΔK_A and ΔK_C are obtained based on the models in Newmann & Raju [22] and Smith & Hurworth [23].

The sum of the membrane stresses, σ_t and the bending stresses, σ_b is taken as

$$\sigma_t + \sigma_b = \Delta\sigma \quad (16)$$

It is assumed that the ratio between bending and membrane stresses is η , implying that

$$\sigma_t = \frac{1}{\eta+1} \Delta\sigma \quad \text{and} \quad \sigma_b = \frac{\eta}{\eta+1} \Delta\sigma \quad (17)$$

Load shedding (linear moment release) is considered in accordance with the formulation proposed in Aghaakouchak et al. [24].

The stress range $\Delta\sigma$ is obtained from

$$\Delta\sigma = Z_{wave} Z_{SCF} Y \Delta\sigma^e \quad (18)$$

where

Z_{wave} and Z_{SCF} model uncertainties

Y model uncertainty related to geometry function

$\Delta\sigma^e$ equivalent stress range:

$$\Delta\sigma^e = \left[\frac{1}{n} \sum_{i=1}^{n_\sigma} n_i \Delta\sigma_i^m \right]^{1/m} \quad (19)$$

The total number of stress ranges per year, n is

$$n = \sum_{i=1}^{n_\sigma} n_i \quad (20)$$

In the assessment of the equivalent constant stress range the effect of a possible lower threshold value ΔK_{TH} on the crack growth inducing stress intensity factor ΔK has not been taken into account explicitly. This effect is assumed implicitly accounted for by evaluation of (19) using the appropriate SN-curve exponent m .

The crack initiation time N_I is modeled as Weibull distributed with expected value μ_0 and coefficient of variation equal to 0.35, see e.g. Lassen [25].

The limit state function is written

$$g(\mathbf{x}) = N - nt \quad (21)$$

where t is time in the interval from 0 to the service life T_L .

In order to model the effect of different weld qualities, two different values of the crack depth at initiation a_0 can be used: 0.1 mm and 0.5 mm corresponding approximately to high and low material control. The corresponding assumed length c_0 is 5 times the crack depth. The critical crack depth a_c is taken as the thickness of the tubular member. The probabilistic modelling used in the fracture mechanical reliability analysis is shown in Table 3.

The parameters $\mu_{\ln c_c}$ and μ_0 are now fitted such that difference between the probability distribution functions for the fatigue live determined using the SN-approach and the fracture mechanical approach is minimized as illustrated in the example below.

Alternatively, or in addition to the above modeling the initial crack length can be modeled as a stochastic variable, for example by an exponential distribution function, and the crack initiation time N_I can be neglected.

Variable	Dist.	Expected value	Standard deviation
N_I	W	μ_0 (reliability based fit to SN approach)	$0.35 \mu_0$
a_0	D	0.1 mm (high material control) / 0.5 mm (low material control)	
$\ln C_C$	N	$\mu_{\ln C_C}$ (reliability based fit to SN approach)	0.77
m	D	m -value corresponding to the low cycle part of the bi-linear SN-curve	
Z_{SCF}	LN	1	0 / 0.05 / ... / 0.20
Z_{wave}	LN	1	0.10 / 0.15 / 0.30
n	D	Total number of stress ranges per year	
a_c	D	T (thickness)	
η	D	2 / 4	
Y	LN	1	0.1
T	D	10 mm / 30 mm / 50 mm / 100 mm	
T_L	D	20 years / 25 years	
T_F	D	= $FDF \cdot T_L = 25 / 50 / \dots / 250$ years	
$\ln C_C$ and N_I are correlated with correlation coefficient $\rho_{\ln(C_C), N_I} = -0.5$			

Table 3. Uncertainty modelling used in the fracture mechanical reliability analysis. D: Deterministic, N: Normal, LN: LogNormal, W: Weibull.

2.6 Implementation of Generic Inspection Planning

2.6.1 *iPlan*

As an example of implementation of Generic Inspection planning the following generic parameters are selected in [14]:

- COV_{wave} (0.10 / 0.15)
- COV_{SCF} (and associated SN-curve: 0.00 / 0.05 / ... / 0.20)
- a_0 (0.1 mm)
- thickness T (10mm / 50mm / 100mm)
- inspection type and associated POD curve (MPI below water)
- Service life: T_L (25 / 40 years)
- Fatigue life time: T_F (= $FDF \cdot T_L$) ($FDF = 1/3/5/10/15$)
- Degree of Bending (DoB = $1/(1+1/\eta)$) = 0 / 0.8)
- The maximum annual probability of fatigue failure ΔP_F^{\max} ($10^{-2} / 10^{-3} / 3 \cdot 10^{-4} / 10^{-4} / 3 \cdot 10^{-5} / 10^{-5}$)

For practical application of the methodology an Excel spreadsheet iPlan is developed, see also [14] and Straub [21]. iPlan can be used to obtain inspection plans for given input parameters within the range of the above generic values. iPlan is based on interpolation between inspection plans for all these combinations. Each generic inspection plan is obtained as described in the previous sections.

2.6.2 Inspection Planning of Jackets

The present section gives a description of how the generic procedures are incorporated in the inspection planning of the jackets, see [14].

The basis for the inspection planning is a deterministic “single wave” fatigue analysis of the jackets. The analysis includes:

- A standard beam FE model of the jackets.
- The average number of annual waves is grouped in 1 m wave height intervals from 8 compass directions. Each of these waves are stepped through the structure to generate nominal stress ranges in all elements in the jacket. Stokes 5th order wave theory is applied to calculate the kinematics.

The default value of the allowable probability of failure is 10^{-5} . If a higher value is to be applied, a pushover analysis is performed to determine the *RIF* value for the actual detail. A 50 year Stokes 5th order wave is used. Several loading directions may be needed to analyzed to determine the direction giving the lowest *RIF* value. The *RIF* value is converted to an allowable probability of failure as follows:

$$RIF \leq 0.60: \quad P_f = 10^{-5} \text{ (LOG}(P_f) = -5)$$

$$RIF \geq 0.90: \quad P_f = 10^{-3} \text{ (LOG}(P_f) = -3)$$

$$0.60 < RIF < 0.90: \quad \text{Linear variation of LOG}(P_f)$$

The above conversion between *RIF* and P_f is a conservative approximation of the curve in Figure 2 covering that several fatigue critical joints may be present in the same jacket at the same time.

Based on the above a typical fatigue/inspection planning analysis of a jacket may look as follows:

1. Perform a deterministic fatigue analysis
2. Check if the joint/detail is inspection free using the closed form expression for *FDF* and a allowable probability of failure = 10^{-5} (fatigue life $\geq FDF \cdot$ service life)
3. For details which are not inspection free perform a pushover analysis (to determine the allowable probability of failure) and/or reduce the value of COV_{SCF} by a detailed FE analysis of the detail. Check if the joint is inspection free in the service period, ref. point no. 2.
4. For joints which are not inspection free, determine in-service inspections using the iPlan data-base.

In case of modifications of the structure related to changed loads and structural changes the inspection planning should be updated accounting for accumulated fatigue damage, as described in Sørensen et al. [9].

2.7 Examples

2.7.1 Example 1.1

A steel jacket structure installed in year 2000 with service life $T_L=40$ years and located in the southern part of the North Sea is considered. The characteristics for a representative selection of fatigue sensitive details are shown in Table 4, see [14]. In section 4 examples are shown where COV_{SCF} is increased to 30%.

Case	COV_{wave}	COV_{SCF}	SN-curve	T [mm]	T_F [year]	DoB
1	0.1	0.15	T	50	100	0.6
2	0.1	0.10	T	50	100	0.6
3	0.1	0.15	T	30	100	0.6
4	0.1	0.15	T	50	200	0.6
5	0.1	0.15	T	50	100	0.3
6	0.1	0.10	F	50	100	0.6
7	0.1	0.05	F	50	100	0.6
8	0.1	0.10	F	30	100	0.6
9	0.1	0.10	F	50	200	0.6
10	0.1	0.10	F	50	100	0.3

Table 4. Example 1.1 cases.

In Figure 8 the reliability indices (corresponding to accumulated probabilities) for the limit states based the SN-approach and the calibrated fracture mechanics (FM) approach are illustrated for case 1 (see Table 4). It is seen that a very good correspondence is obtained between the two different approaches.

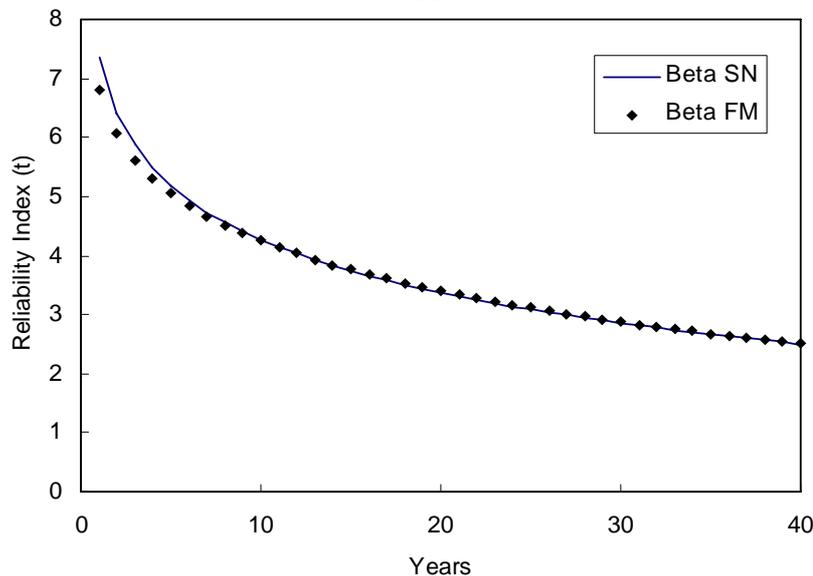


Figure 8. Reliability indices for SN and calibrated Fracture Mechanics corresponding to accumulated probability of failure.

Project
OMAE'03 Example

Date: 2003-01-15
Prepared by: JDS
Checked by: MHF
Approved by: DMS

Inspection plans according to the user-defined thresholds:

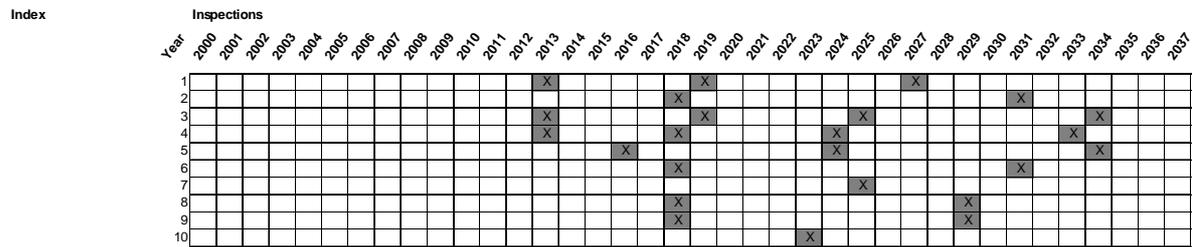


Figure 9. Example 1.1. Inspection plan obtained by iPlan.

Inspection no.	1	2	3	4
Case 1	13	6	8	
Case 2	18	13		
Case 3	13	6	6	9
Case 4	13	5	6	9
Case 5	16	8	10	
Case 6	18	13		
Case 7	25			
Case 8	18	11		
Case 9	18	11		
Case 10	23			

Table 5. Example 1.1. Inspection intervals in years.

In Figure 9 and table 5 the resulting inspection plans obtained by iPlan are shown corresponding to a maximum acceptable annual probability of failure equal to $\Delta P_F^{\max} = 10^{-5}$.

It is seen that

- COV_{SCF} , fatigue life T_F and DoB are important for the inspection plan. Reducing the uncertainty of the stress ranges or extending the design fatigue life increase the year of the first inspection and can reduce the number of inspections.
- After the first inspection there is a tendency that the inspection times become longer with time.

2.7.2 Example 1.2

Case	COV_{wave}	COV_{SCF}	SN-curve	T [mm]	T_F [year]	DoB
Case 1	0.1	0.15	T	20	100	0.6
Case 2	0.1	0.15	T	20	120	0.6
Case 3	0.1	0.15	T	20	140	0.6
Case 4	0.1	0.15	T	20	160	0.6
Case 5	0.1	0.15	T	20	180	0.6
Case 6	0.1	0.15	T	20	200	0.6

Table 6. Example 1.2 cases.

The same jacket structure as in example 1.1 is considered. The characteristics for some fatigue sensitive details are shown in Table 6. The resulting inspection intervals are shown in Table 7. It is seen that

- The time to first inspection increases with FDF
- After the first inspection, the inspection time intervals generally increase with time, but for low FDF s it decrease in the first part of the design lifetime

Inspection no.	1	2	3	4	5
Case 1 - $FDF = 2.5$	13	6	5	7	9
Case 2 - $FDF = 3.0$	16	7	7	9	
Case 3 - $FDF = 3.5$	19	9	9		
Case 4 - $FDF = 4.0$	22	10			
Case 5 - $FDF = 4.5$	25	12			
Case 6 - $FDF = 5.0$	28				

Table 7. Example 1.2. Inspection intervals in years.

2.7.3 Example 2

In this example a simplified one-dimensional crack growth model is used. A steel jacket structure installed in year 2000 with service life $T_L=40$ years and located in the southern part of the North Sea is considered. The characteristics for 4 representative cases are shown in Table 8. Eddy current inspection is used.

Variable	Dist.	Expected value	Standard deviation
N_I	W	Case 1: $\mu_0 = 60 \times 5.7 \cdot 10^6$ (60 years)	$0.35 \mu_0$
	W	Case 2: $\mu_0 = 3 \times 5.7 \cdot 10^6$ (3 years)	$0.35 \mu_0$
	W	Case 3: $\mu_0 = 60 \times 5.7 \cdot 10^6$ (60 years)	$0.35 \mu_0$
	W	Case 4: $\mu_0 = 3 \times 5.7 \cdot 10^6$ (3 years)	$0.35 \mu_0$
a_0	D	Case 1: 1 mm	
	E	Case 2: 0.1 mm	
	E	Case 3: 0.1 mm	
	E	Case 4: 0.1 mm	
$\ln C$	N	Case 1: -25.9	0.77
	N	Case 2: -26.1	0.77
	N	Case 3: -25.3	0.77
	N	Case 4: -27.1	0.77
m	D	3	
Z_{SCF}	LN	1	0.20
Z_{wave}	LN	1	0.10
n	D	$5.7 \cdot 10^6$ per year	
a_c	D	T (thickness)	
Y	LN	1	0.1
T	D	50 mm	
T_F	D	$= FDF T_L = 160$ years	
$\ln C$ and N_I are correlated with correlation coefficient $\rho_{\ln(C),N_I} = -0.5$			

Table 8. Example 2. Uncertainty modelling used in the fracture mechanical reliability analysis. D: Deterministic, N: Normal, LN: LogNormal, W: Weibull, E: Exponential.

In Table 9 the resulting inspection plans obtained are shown corresponding to a maximum acceptable annual probability of failure equal to $\Delta P_F^{\max} = 10^{-4}$ and 10^{-5} . It is seen that

- The inspection time intervals in general are increasing with time - as in example 1
- In case 3 the inspection time intervals are decreasing with time. This is probably due to the large expected value of the initiation time N_I .

In Figures 10-13 are shown the annual probability of failure as function of time before and after inspections.

Inspection no.	FDF	μ_0 years	a_0 mm	ΔP_F^{\max}	1	2	3	4	5
Case 1	4	60	D(1)	10^{-4}	15	7	8		
Case 2	4	3	E(0.1)	10^{-4}	10	4	5	6	7
Case 3	4	60	E(0.1)	10^{-4}	21	6	6	5	
Case 4	4	3	E(0.1)	10^{-5}	16	5	5	6	7

Table 9. Example 2. Inspection intervals in years.

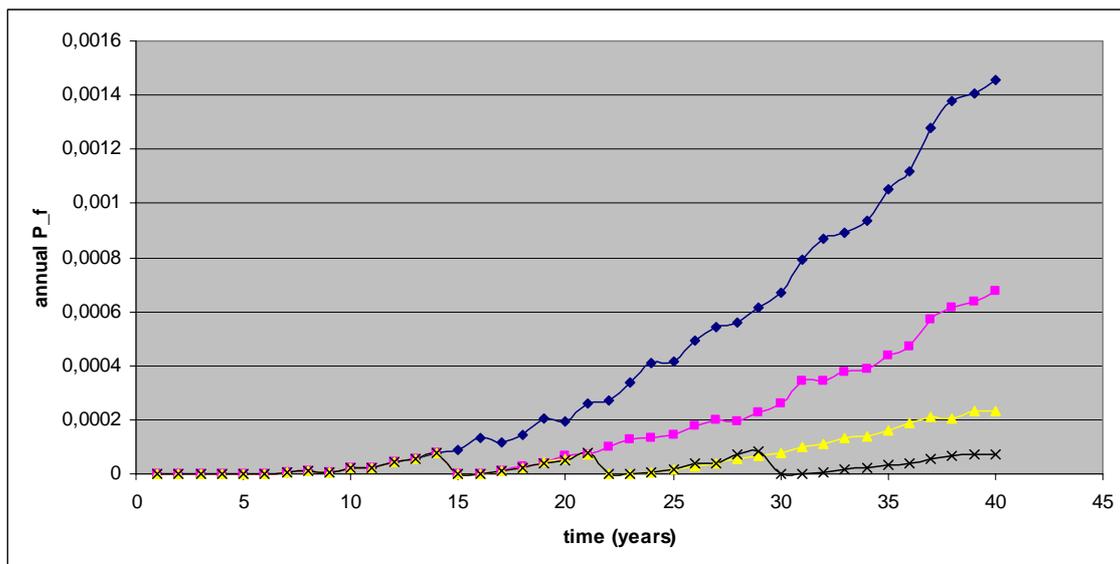


Figure 10. Example 2 – case 1. Annual probability of failure as function of time before and after inspections. $\Delta P_F^{\max} = 10^{-4}$.

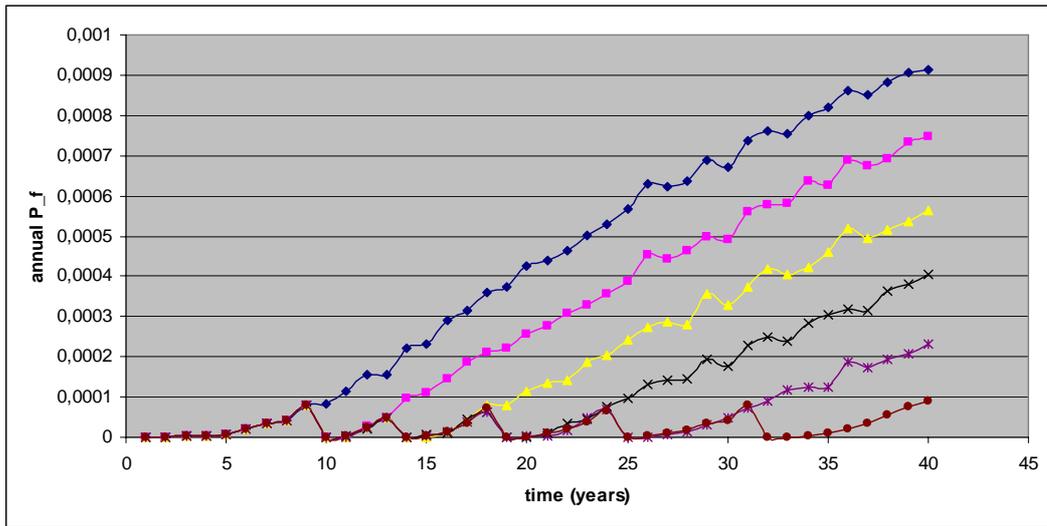


Figure 11. Example 2 – case 2. Annual probability of failure as function of time before and after inspections. $\Delta P_F^{\max} = 10^{-4}$.

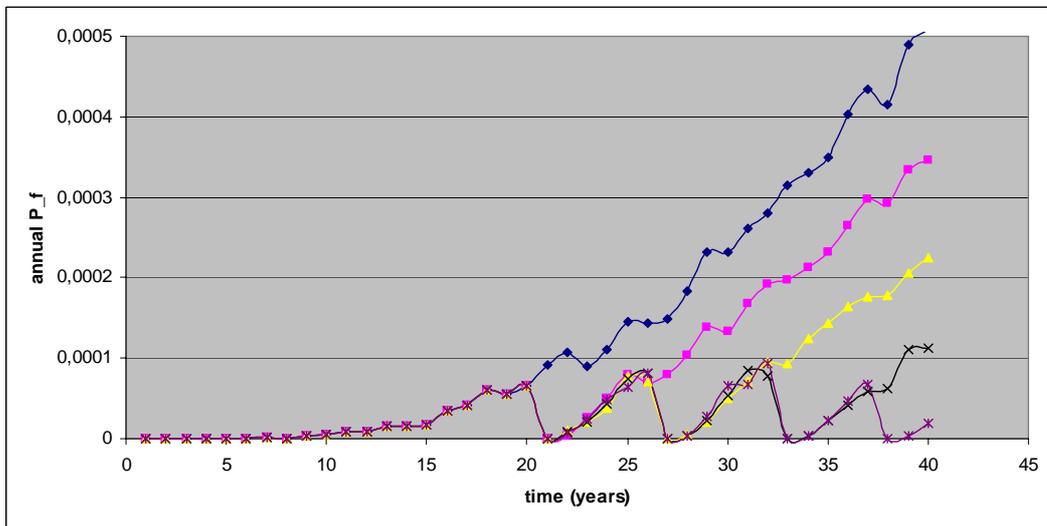


Figure 12. Example 2 – case 3. Annual probability of failure as function of time before and after inspections. $\Delta P_F^{\max} = 10^{-4}$.

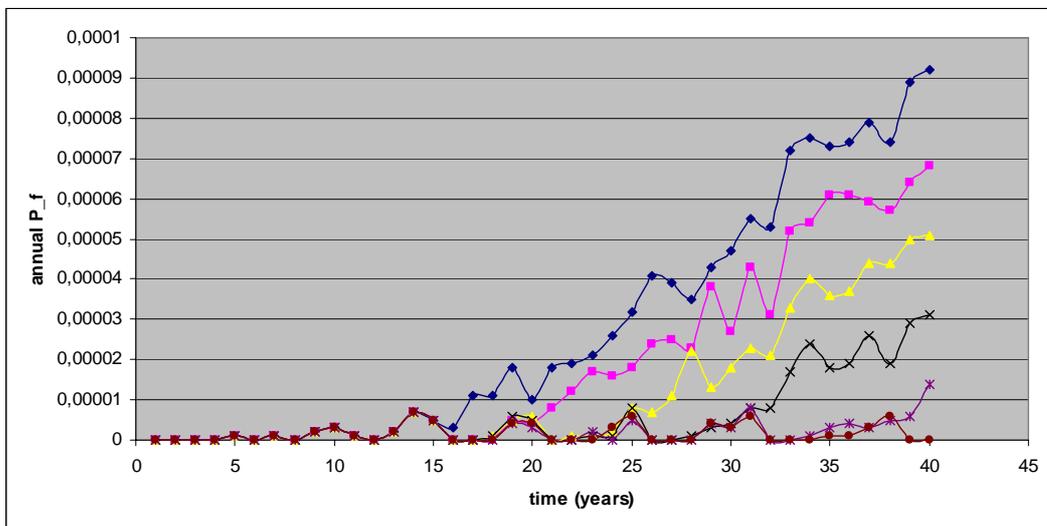


Figure 13. Example 2 – case 4. Annual probability of failure as function of time before and after inspections. $\Delta P_F^{\max} = 10^{-5}$.

3 Inspection planning and systems effects for older installations – Platforms

In the next sections are described various investigations in reliability-based inspection planning with the aim to discuss and investigate how increased inspection time intervals could be obtained when time approaches the design lifetime – this is intuitively what should be expected but as seen in section 2, traditional reliability-based inspection techniques normally result in increasing inspection time intervals.

Two computer programs for reliability analysis with SN and fracture mechanics (FM) approaches are prepared based on a simulation approach to estimate probabilities of failure. The programs are used in the examples in section 4.

The following observations are included in the considerations for a modified method for reliability-based inspection planning for older installations:

- For aging platform several small cracks are observed – implying an increased risk for crack initiation (and coalescence of small cracks) and growth – thus modelling a bath-tub effect
- Repair of cracks can imply weakening of the material, implying subsequent crack initiation and growth
- Observed cracks can be divided in cracks due to fabrication defects and fatigue growing cracks:
 - Fabrication cracks should have been detected by fabrication control and/or an initial inspections, and are therefore not considered in the following
 - Growing fatigue cracks possibly to be detected by inspections – typically 10% (of welds) is inspected and from these 5% have cracks (defects)

In section 4 is considered the following models for changing inspection intervals for older platforms:

- a. Increase of expected value of initial crack size with time – due to coalescence of smaller cracks
- b. Non-perfect repairs - by detection of cracks the repair is not perfect, and a new crack is initiated
- c. Human errors in inspections (beyond uncertainty included in *POD*-curves)
- d. Increased rate of crack initiation - adjustment of the crack initiation time such that initiation of cracks increase with time (bath tub effect).
- e. The increase of crack initiation can be in excess of the crack initiation expected at the design state (and obtained by reliability-based calibration to SN-curves) due to the aging effects (e.g. by coalescence of small cracks)

In case of lifetime extension the above effects also applies in the extended lifetime.

Representative examples are used to evaluate the different models.

In section 5 the following system effects are considered:

- The assessment of the acceptable annual fatigue probability of failure for a particular component should take into account that there can be a number of fatigue critical components in a structure.
- Due to common loading, common model uncertainties and correlation between inspection qualities it can be expected that information obtained from inspection of

- one component can be used not only to update the inspection plan for that component, but also for other nearby components.
- In some cases the development of a crack in one component causes a stiffness reduction which imply that loads are redistributed and thereby increase the stress ranges in some of the other fatigue critical details.

It is noted that a basic assumption in the reliability-based inspection planning approach used in this report is that a Bayesian approach can be used. This implies that probabilities of failure can be updated in a consistent way when new information becomes available. The Bayesian approach is also consistent with rational risk analysis and decision making based on the framework of pre-posterior analysis from classical Bayesian decision theory see e.g. Raiffa and Schlaifer [19] and Benjamin and Cornell [20] and implemented as described in e.g. Sørensen et al. [6]. This basic assumption is also very important to understand why longer inspection time intervals are obtained when no finds at the inspections are assumed.

4 Inspection planning for older - modified models for stochastic parameters

The basic assumption in the RBI approach described in section 2 is that in a fatigue critical detail a crack initiate at some time modelled by a stochastic variable, see figure 14. However, it is frequently observed that damage initiation rates follow a bath-tub form, see figure 15. Initial damages are mainly due to fabrication / construction defects, and at the end of the expected lifetime the damage rate increase. In figures 16 and 17 combined models are illustrated where the ‘bath-tub’ effect is combined with the ‘usual’ crack initiation model.

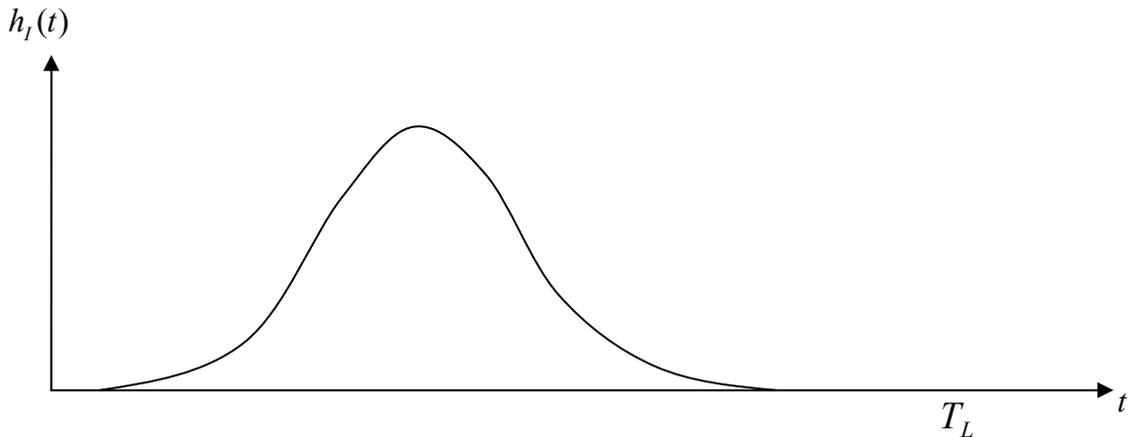


Figure 14. Basic model for crack initiation time.

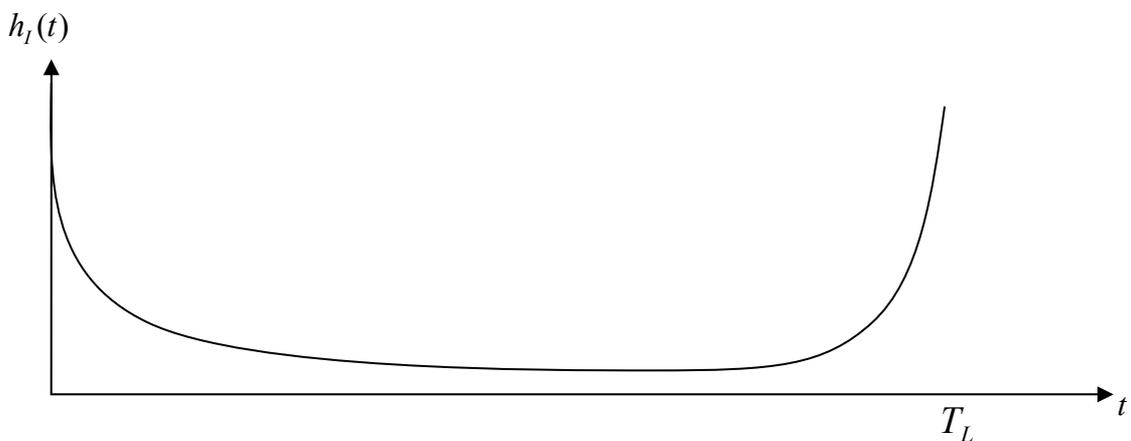


Figure 15. Bath-tub model for damage initiation.

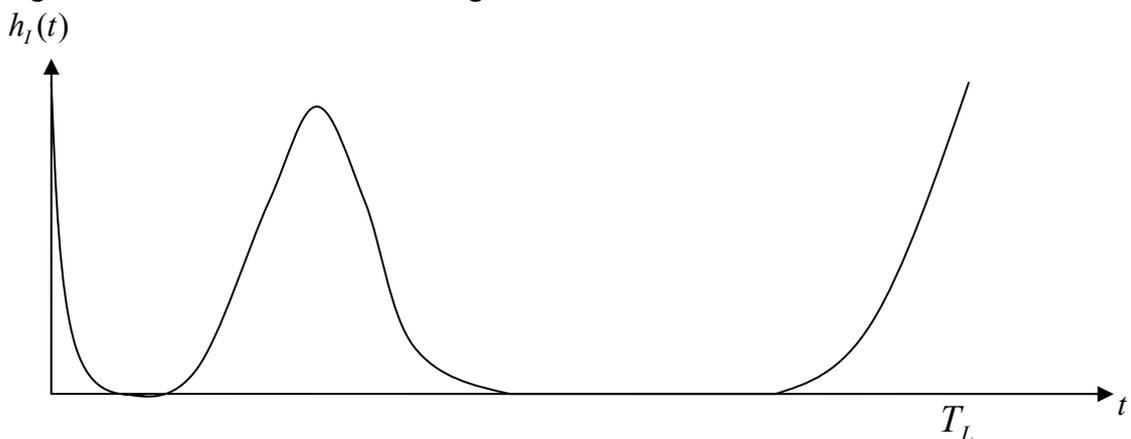


Figure 16. A combined model for damage initiation including initial defects.

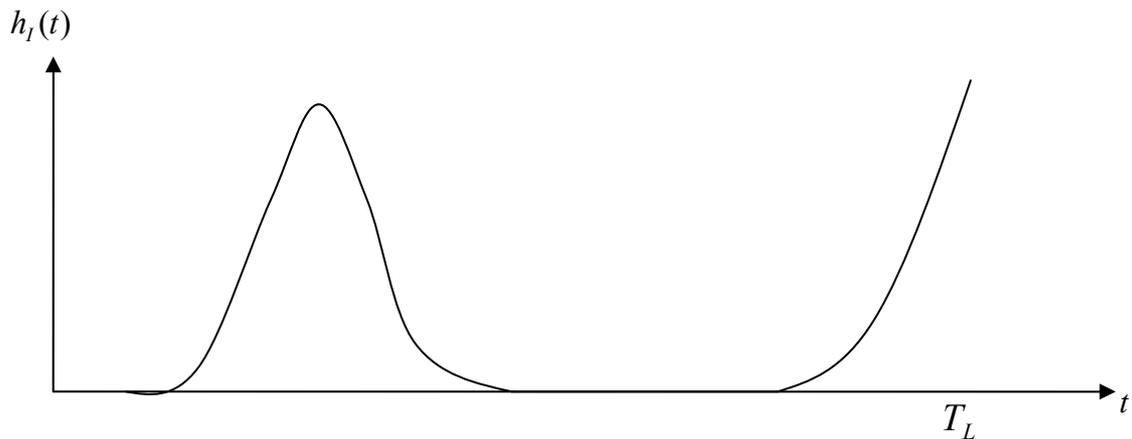


Figure 17. A combined model for damage initiation without initial defects.

In the following four different models are investigated, where the stochastic model is modified with the objective to improve the modelling of the behaviour of aging offshore structures with welded steel details. The models in figures 14 and 17 will be investigated by examples.

4.1 Modified stochastic models for older structures

Model a) Increase of initial cracks with time

The initial crack size a_0 increases with time. This can be due to increased crack coalescence, material weakening, ... By increasing the initial crack size when the structure becomes older, it can be expected that the inspection time intervals decrease. This can be modelled by the model in figure 18 for the expected value of a_0 .

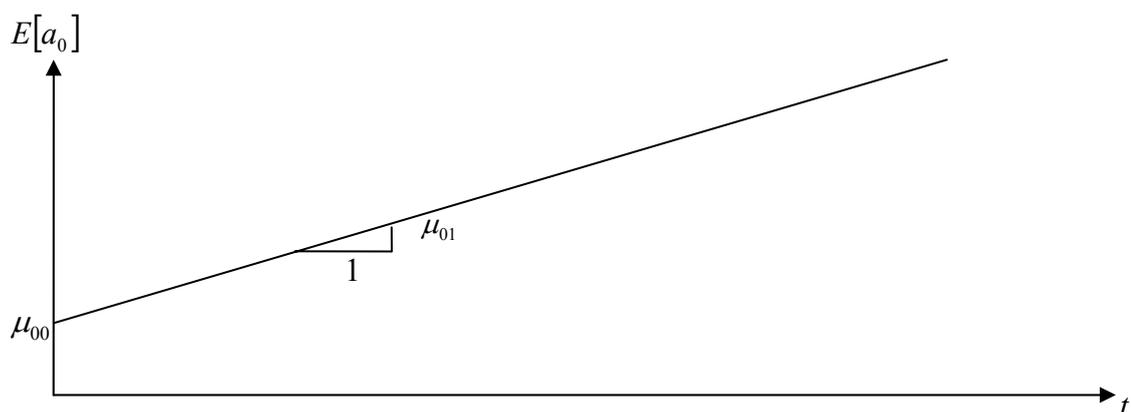


Figure 18. Model for increased initial crack size.

The parameters values could e.g. be: $\mu_{00} = 0.4$ mm and $\mu_{01} = 0.05$ mm/year.

The time to crack initiation N_f is assumed Weibull distributed with expected value μ_{f0} and $COV = 0.35$.

Model b) Non-perfect repairs

It is assumed that the repairs are non-perfect, e.g. due to weakening of the material in connection with the repair. Therefore in case of an inspection and detection (and repair)

of a crack, it is assumed that a new crack is initiated immediately after the repair ($N_I = 0$) with

- expected value of initial crack size equal to μ_{00} (as in model a) – independent on the former initial crack sizes
- X_{SCF} , X_{wave} and Y (defined in section 2.5.2) fully correlated with corresponding parameters before repair
- $\ln C$ (defined in section 2.5.2) statistically independent on $\ln C$ before repair

Non-perfect repairs could be expected to have the effect that the inspection time intervals decrease.

Model c) Human errors in inspections

It is assumed that gross/human errors can occur in connection with the inspections, including that the inspection is omitted erroneously. These errors are beyond the uncertainty included in the *POD* curves. The probability of a human error is assumed to be P_{HE} and if a human error occur then a crack is not detected.

Human errors causing that less critical cracks are detected could be expected to have the effect that the inspection time intervals decrease.

Model d) Initiation of extra cracks

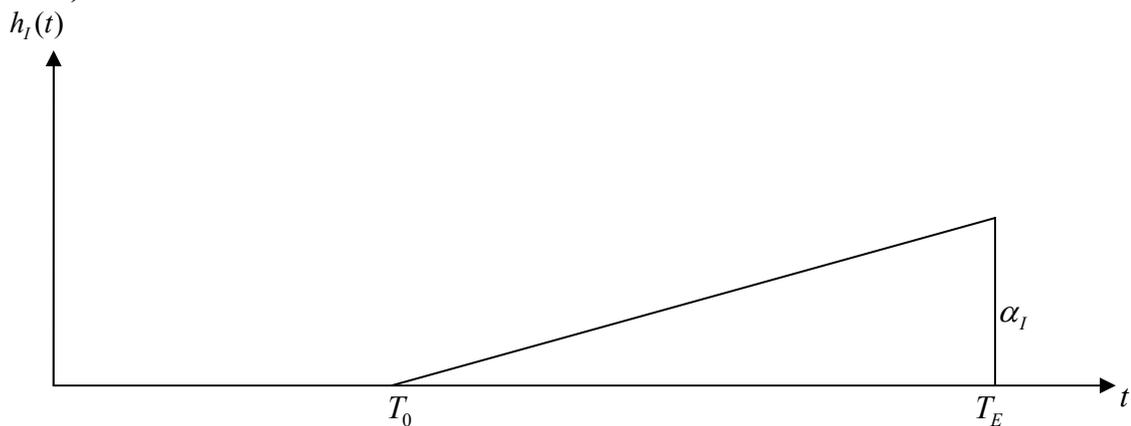


Figure 19. Initiation rate of extra cracks – linear model.

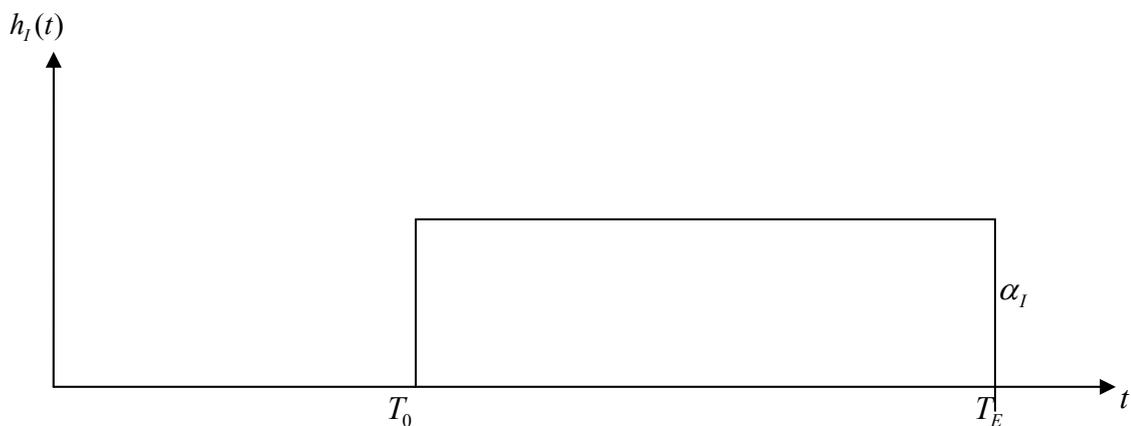


Figure 20. Initiation rate of extra cracks – constant model.

In this model it is assumed that more cracks initiate when time is approaching the design lifetime (due to weakening by age effects) than assumed in the initial calibration of the fracture mechanics model. These models correspond to the model in figure 17.

The extra cracks are assumed to initiate following a linear or a constant model in the time interval $[T_0, T_E]$, see figures 19 and 20. Extra new cracks could be expected to have the effect that the inspection time intervals decrease.

4.2 Examples

Computer programs using Monte Carlo simulations have been programmed to estimate the reliability as function of time by the SN-approach and by the fracture mechanics (FM). In order to reduce the computational effort a 1-dimensional fracture mechanics model is used. It is expected that for the examples considered the same principal behaviour of the resulting inspection plans will be obtained as if a 2-dimensional crack growth model was used. The stochastic models used are shown in tables 10 and 11, see also section 2.

Variable	Distribution	Expected value	Standard deviation
Δ	LN	1	0.2
X_{SCF}	LN	1	$COV_{SCF} = 0.10$
X_{wave}	LN	1	$COV_{wave} = 0.30$
T_F	D	75 years	
T_L	D	25 years	
m_1	D	3	
$\log K_1$	N	12.048	0.218
m_2	D	4	
$\log K_2$	N	13.980	0.291
log K_1 and log K_2 are assumed fully correlated			

Table 10. Stochastic model for SN-approach.

Variable	Dist.	Expected value	Standard deviation
N_I	W	μ_{I0} (fitted)	$0.35 \mu_{I0}$
a_0	D	0.4 mm	
$\ln C_C$	N	$\mu_{\ln C_C}$ (fitted)	0.77
m	D	3	
X_{SCF}	LN	1	0.10
X_{wave}	LN	1	0.30
n	D	Total number of stress ranges per year	
a_c	D	T (thickness)	
η	D	2 / 4	
Y	LN	1	0.1
T	D	50 mm	
T_L	D	25 years	
T_F	D	= FDF $T_L = 25 / 50 / 75$ years	
ln C_C and N_I are correlated with correlation coefficient $\rho_{\ln(C_C), N_I} = -0.5$			

Table 11. Uncertainty modelling used in the fracture mechanical reliability analysis. D: Deterministic, N: Normal, LN: LogNormal, W: Weibull.

The parameters in the fracture mechanical model are calibrated to

$$\mu_{t_0} = 5 \text{ years}$$

$$\mu_{\ln C_c} = -26.5$$

The reliability index (based on accumulated probability of failure) is shown in figure 21.

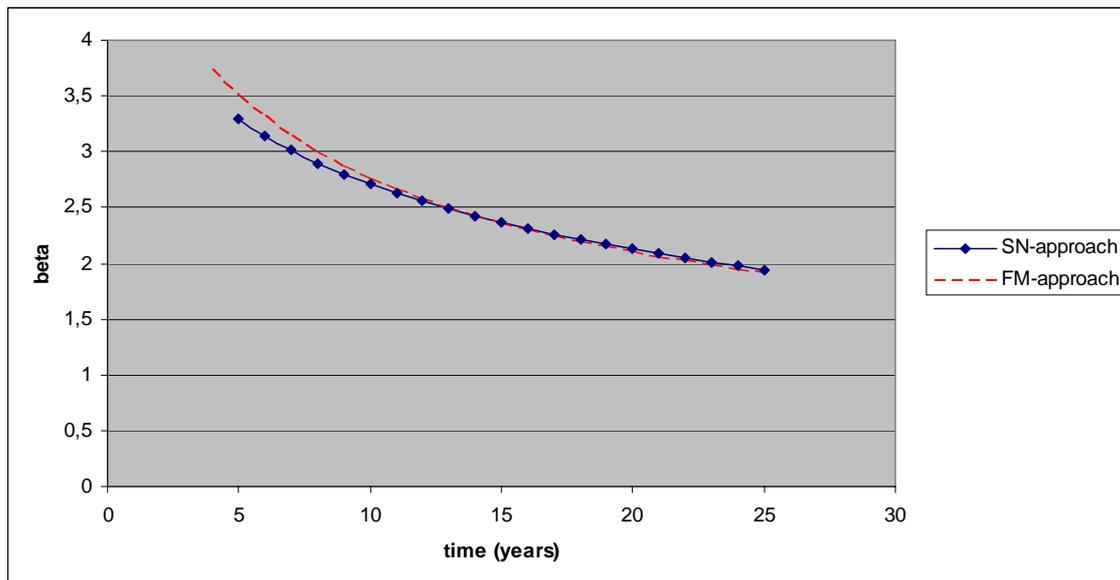


Figure 21. Reliability index (accumulated) as function of time for SN approach and calibrated FM-approach.

The four models for modified stochastic parameters in section 4.1 are investigated in the following sections with maximum acceptable annual probability of failure $\Delta P_F^{\max} = 10^{-4}$.

No modification

The inspection times and time intervals are shown in table 12 with no modifications.

inspection times (upper values) and intervals (lower values) in years								
4	6	9	13	18	24	32	40	
4	2	3	4	5	6	8	8	

Table 12. Inspection times and time intervals – no modification.

Comment:

- The inspection time intervals increase with time – most of the fastest growing cracks are detected and repaired in the first inspections, and thus only few critical cracks are left when time approaches the design lifetime.

Model a) Increase of expected value of initial crack size with time

The inspection times and time intervals are shown in table 13 with increase of expected value of initial crack size with time.

μ_{01} [mm/year]	inspection times (upper values) and intervals (lower values) in years							
0	4	6	9	13	18	24	32	40
	4	2	3	4	5	6	8	8
0.05	3	5	8	11	16	22	30	43
	3	2	3	3	5	6	8	13
0.10	3	5	7	11	16	22	31	45
	3	2	2	4	5	6	9	14

Table 13. Inspection times and time intervals – Increase of expected value of initial crack size with time. $\mu_{00}=0.4$ mm.

Comment:

- The inspection time intervals still increase with time – slightly slower in the beginning but later the time intervals become longer.

Model b) Non-perfect repairs

The inspection times and time intervals are shown in table 14 with non-perfect repairs.

	inspection times (upper values) and intervals (lower values) in years								
No modification	4	6	9	13	18	24	32	40	
	4	2	3	4	5	6	8	8	
New cracks after repair	4	6	8	11	15	21	29	38	49
	4	2	2	3	4	6	8	9	11

Table 14. Inspection times and time intervals – Non-perfect repairs.

Comment:

- The inspection time intervals still increase with time – but the increase is slower with non-perfect repairs.

Model c) Human errors in inspections

The inspection times and time intervals are shown in table 15 when human errors are included in the inspections.

P_{HE}	inspection times (upper values) and intervals (lower values) in years									
0	4	6	9	13	18	24	32	40		
	4	2	3	4	5	6	8	8		
0.05	4	6	8	11	15	21	29	38	49	
	4	2	2	3	4	6	8	9	11	
0.10	4	6	8	11	15	21	27	35	45	
	4	2	2	3	4	6	6	8	10	
0.15	4	5	7	9	11	14	19	24	32	41
	4	1	2	2	2	3	5	6	8	9

Table 15. Inspection times and time intervals – Human errors in inspections.

Comment:

- The inspection time intervals still increase with time – but the increase is becoming smaller with increasing probability of human error.

Model d) Initiation of more cracks due to age effects

Extra cracks are assumed to initiate in the time interval $[T_0, T_E]$, see models in figures 19 and 20. In this example it is assumed that the extra number of cracks is equal to the number of ‘ordinary’ cracks.

The number of simulations are increased compared to model a) – c). therefore the inspection times are slightly changed. Inspection times and time intervals are shown in table 16 for different models and values of T_0 and T_F and $\Delta P_F^{\max} = 10^{-4}$. Table 17 shows results with maximum acceptable annual probability of failure $\Delta P_F^{\max} = 10^{-3}$.

Model	α_I	Inspection times (upper values) and intervals (lower values) in years																																																																																																																																																																																																																																																																																																			
No extra cracks		4	6	9	12	17	23	32	41							4	2	3	3	5	6	9	9					Constant [15 ; 50]	$1 \times 1/35$	4	5	7	9	12	16	22	30	38	48					4	1	2	2	3	4	6	8	8	10			Constant [12 ; 50]	$1 \times 1/38$	4	5	7	9	12	16	21	28	36	45					4	1	2	2	3	4	5	7	8	9			Constant [10 ; 50]	$1 \times 1/40$	4	5	7	9	12	16	21	27	34	40					4	1	2	2	3	4	5	6	7	6			Linear [15 ; 50]	$1 \times 2/35$	4	5	7	9	12	16	22	29	38	47					4	1	2	2	3	4	6	7	9	9			Linear [15 ; 25]	$1 \times 2/10$	4	5	7	9	12	16	22	28	34	41					4	1	2	2	3	4	6	6	6	7			Constant [15 ; 25]	$1 \times 1/10$	4	5	7	9	12	16	21	27	34	40	49				4	1	2	2	3	4	5	6	7	6	9		Constant [10 ; 25]	$1 \times 1/15$	4	5	7	9	12	16	21	27	34	40	49				4	1	2	2	3	4	5	6	7	6	9																Linear [10 ; 25]	$2 \times 2/15$	4	5	7	9	12	16	20	25	32	38	48				4	1	2	2	3	4	4	5	7	6	10		Linear [10 ; 25]	$3 \times 2/15$	4	5	7	9	11	15	20	24	28	32	39	46			4	1	2	2	2	4	5	4	4	4	7	7
		4	2	3	3	5	6	9	9					Constant [15 ; 50]	$1 \times 1/35$	4	5	7	9	12	16	22	30	38	48					4	1	2	2	3	4	6	8	8	10			Constant [12 ; 50]	$1 \times 1/38$	4	5	7	9	12	16	21	28	36	45					4	1	2	2	3	4	5	7	8	9			Constant [10 ; 50]	$1 \times 1/40$	4	5	7	9	12	16	21	27	34	40					4	1	2	2	3	4	5	6	7	6			Linear [15 ; 50]	$1 \times 2/35$	4	5	7	9	12	16	22	29	38	47					4	1	2	2	3	4	6	7	9	9			Linear [15 ; 25]	$1 \times 2/10$	4	5	7	9	12	16	22	28	34	41					4	1	2	2	3	4	6	6	6	7			Constant [15 ; 25]	$1 \times 1/10$	4	5	7	9	12	16	21	27	34	40	49				4	1	2	2	3	4	5	6	7	6	9		Constant [10 ; 25]	$1 \times 1/15$	4	5	7	9	12	16	21	27	34	40	49				4	1	2	2	3	4	5	6	7	6	9																Linear [10 ; 25]	$2 \times 2/15$	4	5	7	9	12	16	20	25	32	38	48				4	1	2	2	3	4	4	5	7	6	10		Linear [10 ; 25]	$3 \times 2/15$	4	5	7	9	11	15	20	24	28	32	39	46			4	1	2	2	2	4	5	4	4	4	7	7														
Constant [15 ; 50]	$1 \times 1/35$	4	5	7	9	12	16	22	30	38	48					4	1	2	2	3	4	6	8	8	10			Constant [12 ; 50]	$1 \times 1/38$	4	5	7	9	12	16	21	28	36	45					4	1	2	2	3	4	5	7	8	9			Constant [10 ; 50]	$1 \times 1/40$	4	5	7	9	12	16	21	27	34	40					4	1	2	2	3	4	5	6	7	6			Linear [15 ; 50]	$1 \times 2/35$	4	5	7	9	12	16	22	29	38	47					4	1	2	2	3	4	6	7	9	9			Linear [15 ; 25]	$1 \times 2/10$	4	5	7	9	12	16	22	28	34	41					4	1	2	2	3	4	6	6	6	7			Constant [15 ; 25]	$1 \times 1/10$	4	5	7	9	12	16	21	27	34	40	49				4	1	2	2	3	4	5	6	7	6	9		Constant [10 ; 25]	$1 \times 1/15$	4	5	7	9	12	16	21	27	34	40	49				4	1	2	2	3	4	5	6	7	6	9																Linear [10 ; 25]	$2 \times 2/15$	4	5	7	9	12	16	20	25	32	38	48				4	1	2	2	3	4	4	5	7	6	10		Linear [10 ; 25]	$3 \times 2/15$	4	5	7	9	11	15	20	24	28	32	39	46			4	1	2	2	2	4	5	4	4	4	7	7																												
		4	1	2	2	3	4	6	8	8	10			Constant [12 ; 50]	$1 \times 1/38$	4	5	7	9	12	16	21	28	36	45					4	1	2	2	3	4	5	7	8	9			Constant [10 ; 50]	$1 \times 1/40$	4	5	7	9	12	16	21	27	34	40					4	1	2	2	3	4	5	6	7	6			Linear [15 ; 50]	$1 \times 2/35$	4	5	7	9	12	16	22	29	38	47					4	1	2	2	3	4	6	7	9	9			Linear [15 ; 25]	$1 \times 2/10$	4	5	7	9	12	16	22	28	34	41					4	1	2	2	3	4	6	6	6	7			Constant [15 ; 25]	$1 \times 1/10$	4	5	7	9	12	16	21	27	34	40	49				4	1	2	2	3	4	5	6	7	6	9		Constant [10 ; 25]	$1 \times 1/15$	4	5	7	9	12	16	21	27	34	40	49				4	1	2	2	3	4	5	6	7	6	9																Linear [10 ; 25]	$2 \times 2/15$	4	5	7	9	12	16	20	25	32	38	48				4	1	2	2	3	4	4	5	7	6	10		Linear [10 ; 25]	$3 \times 2/15$	4	5	7	9	11	15	20	24	28	32	39	46			4	1	2	2	2	4	5	4	4	4	7	7																																										
Constant [12 ; 50]	$1 \times 1/38$	4	5	7	9	12	16	21	28	36	45					4	1	2	2	3	4	5	7	8	9			Constant [10 ; 50]	$1 \times 1/40$	4	5	7	9	12	16	21	27	34	40					4	1	2	2	3	4	5	6	7	6			Linear [15 ; 50]	$1 \times 2/35$	4	5	7	9	12	16	22	29	38	47					4	1	2	2	3	4	6	7	9	9			Linear [15 ; 25]	$1 \times 2/10$	4	5	7	9	12	16	22	28	34	41					4	1	2	2	3	4	6	6	6	7			Constant [15 ; 25]	$1 \times 1/10$	4	5	7	9	12	16	21	27	34	40	49				4	1	2	2	3	4	5	6	7	6	9		Constant [10 ; 25]	$1 \times 1/15$	4	5	7	9	12	16	21	27	34	40	49				4	1	2	2	3	4	5	6	7	6	9																Linear [10 ; 25]	$2 \times 2/15$	4	5	7	9	12	16	20	25	32	38	48				4	1	2	2	3	4	4	5	7	6	10		Linear [10 ; 25]	$3 \times 2/15$	4	5	7	9	11	15	20	24	28	32	39	46			4	1	2	2	2	4	5	4	4	4	7	7																																																								
		4	1	2	2	3	4	5	7	8	9			Constant [10 ; 50]	$1 \times 1/40$	4	5	7	9	12	16	21	27	34	40					4	1	2	2	3	4	5	6	7	6			Linear [15 ; 50]	$1 \times 2/35$	4	5	7	9	12	16	22	29	38	47					4	1	2	2	3	4	6	7	9	9			Linear [15 ; 25]	$1 \times 2/10$	4	5	7	9	12	16	22	28	34	41					4	1	2	2	3	4	6	6	6	7			Constant [15 ; 25]	$1 \times 1/10$	4	5	7	9	12	16	21	27	34	40	49				4	1	2	2	3	4	5	6	7	6	9		Constant [10 ; 25]	$1 \times 1/15$	4	5	7	9	12	16	21	27	34	40	49				4	1	2	2	3	4	5	6	7	6	9																Linear [10 ; 25]	$2 \times 2/15$	4	5	7	9	12	16	20	25	32	38	48				4	1	2	2	3	4	4	5	7	6	10		Linear [10 ; 25]	$3 \times 2/15$	4	5	7	9	11	15	20	24	28	32	39	46			4	1	2	2	2	4	5	4	4	4	7	7																																																																						
Constant [10 ; 50]	$1 \times 1/40$	4	5	7	9	12	16	21	27	34	40					4	1	2	2	3	4	5	6	7	6			Linear [15 ; 50]	$1 \times 2/35$	4	5	7	9	12	16	22	29	38	47					4	1	2	2	3	4	6	7	9	9			Linear [15 ; 25]	$1 \times 2/10$	4	5	7	9	12	16	22	28	34	41					4	1	2	2	3	4	6	6	6	7			Constant [15 ; 25]	$1 \times 1/10$	4	5	7	9	12	16	21	27	34	40	49				4	1	2	2	3	4	5	6	7	6	9		Constant [10 ; 25]	$1 \times 1/15$	4	5	7	9	12	16	21	27	34	40	49				4	1	2	2	3	4	5	6	7	6	9																Linear [10 ; 25]	$2 \times 2/15$	4	5	7	9	12	16	20	25	32	38	48				4	1	2	2	3	4	4	5	7	6	10		Linear [10 ; 25]	$3 \times 2/15$	4	5	7	9	11	15	20	24	28	32	39	46			4	1	2	2	2	4	5	4	4	4	7	7																																																																																				
		4	1	2	2	3	4	5	6	7	6			Linear [15 ; 50]	$1 \times 2/35$	4	5	7	9	12	16	22	29	38	47					4	1	2	2	3	4	6	7	9	9			Linear [15 ; 25]	$1 \times 2/10$	4	5	7	9	12	16	22	28	34	41					4	1	2	2	3	4	6	6	6	7			Constant [15 ; 25]	$1 \times 1/10$	4	5	7	9	12	16	21	27	34	40	49				4	1	2	2	3	4	5	6	7	6	9		Constant [10 ; 25]	$1 \times 1/15$	4	5	7	9	12	16	21	27	34	40	49				4	1	2	2	3	4	5	6	7	6	9																Linear [10 ; 25]	$2 \times 2/15$	4	5	7	9	12	16	20	25	32	38	48				4	1	2	2	3	4	4	5	7	6	10		Linear [10 ; 25]	$3 \times 2/15$	4	5	7	9	11	15	20	24	28	32	39	46			4	1	2	2	2	4	5	4	4	4	7	7																																																																																																		
Linear [15 ; 50]	$1 \times 2/35$	4	5	7	9	12	16	22	29	38	47					4	1	2	2	3	4	6	7	9	9			Linear [15 ; 25]	$1 \times 2/10$	4	5	7	9	12	16	22	28	34	41					4	1	2	2	3	4	6	6	6	7			Constant [15 ; 25]	$1 \times 1/10$	4	5	7	9	12	16	21	27	34	40	49				4	1	2	2	3	4	5	6	7	6	9		Constant [10 ; 25]	$1 \times 1/15$	4	5	7	9	12	16	21	27	34	40	49				4	1	2	2	3	4	5	6	7	6	9																Linear [10 ; 25]	$2 \times 2/15$	4	5	7	9	12	16	20	25	32	38	48				4	1	2	2	3	4	4	5	7	6	10		Linear [10 ; 25]	$3 \times 2/15$	4	5	7	9	11	15	20	24	28	32	39	46			4	1	2	2	2	4	5	4	4	4	7	7																																																																																																																
		4	1	2	2	3	4	6	7	9	9			Linear [15 ; 25]	$1 \times 2/10$	4	5	7	9	12	16	22	28	34	41					4	1	2	2	3	4	6	6	6	7			Constant [15 ; 25]	$1 \times 1/10$	4	5	7	9	12	16	21	27	34	40	49				4	1	2	2	3	4	5	6	7	6	9		Constant [10 ; 25]	$1 \times 1/15$	4	5	7	9	12	16	21	27	34	40	49				4	1	2	2	3	4	5	6	7	6	9																Linear [10 ; 25]	$2 \times 2/15$	4	5	7	9	12	16	20	25	32	38	48				4	1	2	2	3	4	4	5	7	6	10		Linear [10 ; 25]	$3 \times 2/15$	4	5	7	9	11	15	20	24	28	32	39	46			4	1	2	2	2	4	5	4	4	4	7	7																																																																																																																														
Linear [15 ; 25]	$1 \times 2/10$	4	5	7	9	12	16	22	28	34	41					4	1	2	2	3	4	6	6	6	7			Constant [15 ; 25]	$1 \times 1/10$	4	5	7	9	12	16	21	27	34	40	49				4	1	2	2	3	4	5	6	7	6	9		Constant [10 ; 25]	$1 \times 1/15$	4	5	7	9	12	16	21	27	34	40	49				4	1	2	2	3	4	5	6	7	6	9																Linear [10 ; 25]	$2 \times 2/15$	4	5	7	9	12	16	20	25	32	38	48				4	1	2	2	3	4	4	5	7	6	10		Linear [10 ; 25]	$3 \times 2/15$	4	5	7	9	11	15	20	24	28	32	39	46			4	1	2	2	2	4	5	4	4	4	7	7																																																																																																																																												
		4	1	2	2	3	4	6	6	6	7			Constant [15 ; 25]	$1 \times 1/10$	4	5	7	9	12	16	21	27	34	40	49				4	1	2	2	3	4	5	6	7	6	9		Constant [10 ; 25]	$1 \times 1/15$	4	5	7	9	12	16	21	27	34	40	49				4	1	2	2	3	4	5	6	7	6	9																Linear [10 ; 25]	$2 \times 2/15$	4	5	7	9	12	16	20	25	32	38	48				4	1	2	2	3	4	4	5	7	6	10		Linear [10 ; 25]	$3 \times 2/15$	4	5	7	9	11	15	20	24	28	32	39	46			4	1	2	2	2	4	5	4	4	4	7	7																																																																																																																																																										
Constant [15 ; 25]	$1 \times 1/10$	4	5	7	9	12	16	21	27	34	40	49				4	1	2	2	3	4	5	6	7	6	9		Constant [10 ; 25]	$1 \times 1/15$	4	5	7	9	12	16	21	27	34	40	49				4	1	2	2	3	4	5	6	7	6	9																Linear [10 ; 25]	$2 \times 2/15$	4	5	7	9	12	16	20	25	32	38	48				4	1	2	2	3	4	4	5	7	6	10		Linear [10 ; 25]	$3 \times 2/15$	4	5	7	9	11	15	20	24	28	32	39	46			4	1	2	2	2	4	5	4	4	4	7	7																																																																																																																																																																								
		4	1	2	2	3	4	5	6	7	6	9		Constant [10 ; 25]	$1 \times 1/15$	4	5	7	9	12	16	21	27	34	40	49				4	1	2	2	3	4	5	6	7	6	9																Linear [10 ; 25]	$2 \times 2/15$	4	5	7	9	12	16	20	25	32	38	48				4	1	2	2	3	4	4	5	7	6	10		Linear [10 ; 25]	$3 \times 2/15$	4	5	7	9	11	15	20	24	28	32	39	46			4	1	2	2	2	4	5	4	4	4	7	7																																																																																																																																																																																						
Constant [10 ; 25]	$1 \times 1/15$	4	5	7	9	12	16	21	27	34	40	49				4	1	2	2	3	4	5	6	7	6	9																Linear [10 ; 25]	$2 \times 2/15$	4	5	7	9	12	16	20	25	32	38	48				4	1	2	2	3	4	4	5	7	6	10		Linear [10 ; 25]	$3 \times 2/15$	4	5	7	9	11	15	20	24	28	32	39	46			4	1	2	2	2	4	5	4	4	4	7	7																																																																																																																																																																																																				
		4	1	2	2	3	4	5	6	7	6	9																Linear [10 ; 25]	$2 \times 2/15$	4	5	7	9	12	16	20	25	32	38	48				4	1	2	2	3	4	4	5	7	6	10		Linear [10 ; 25]	$3 \times 2/15$	4	5	7	9	11	15	20	24	28	32	39	46			4	1	2	2	2	4	5	4	4	4	7	7																																																																																																																																																																																																																		
														Linear [10 ; 25]	$2 \times 2/15$	4	5	7	9	12	16	20	25	32	38	48				4	1	2	2	3	4	4	5	7	6	10		Linear [10 ; 25]	$3 \times 2/15$	4	5	7	9	11	15	20	24	28	32	39	46			4	1	2	2	2	4	5	4	4	4	7	7																																																																																																																																																																																																																																
Linear [10 ; 25]	$2 \times 2/15$	4	5	7	9	12	16	20	25	32	38	48				4	1	2	2	3	4	4	5	7	6	10		Linear [10 ; 25]	$3 \times 2/15$	4	5	7	9	11	15	20	24	28	32	39	46			4	1	2	2	2	4	5	4	4	4	7	7																																																																																																																																																																																																																																														
		4	1	2	2	3	4	4	5	7	6	10		Linear [10 ; 25]	$3 \times 2/15$	4	5	7	9	11	15	20	24	28	32	39	46			4	1	2	2	2	4	5	4	4	4	7	7																																																																																																																																																																																																																																																												
Linear [10 ; 25]	$3 \times 2/15$	4	5	7	9	11	15	20	24	28	32	39	46			4	1	2	2	2	4	5	4	4	4	7	7																																																																																																																																																																																																																																																																										
		4	1	2	2	2	4	5	4	4	4	7	7																																																																																																																																																																																																																																																																																								

Table 16. Inspection times and time intervals – extra initiation of cracks. $\Delta P_F^{\max} = 10^{-4}$.

Model	α_I	Inspection times (upper values) and intervals (lower values) in years															
Linear [10 ; 25]	$3 \times 2/15$	6	12	21	30	41					6	6	9	9	11		
		6	6	9	9	11											

Table 17. Inspection times and time intervals – extra initiation of cracks. $\Delta P_F^{\max} = 10^{-3}$.

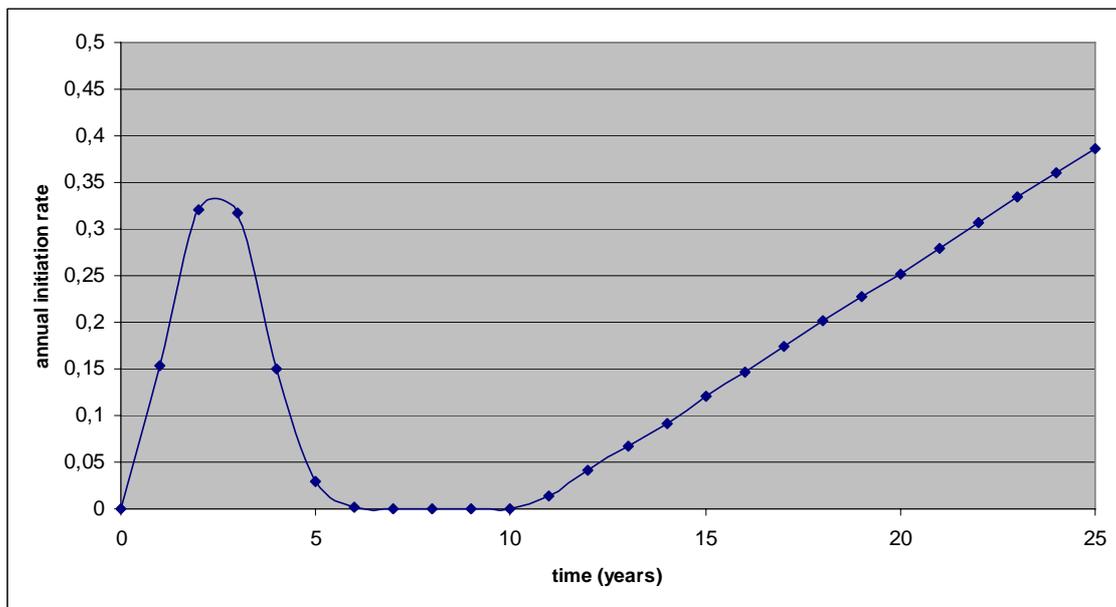


Figure 22. Annual rate of crack initiation - Linear [10 ; 25] and $\alpha_i = 3 \times 2/15$.

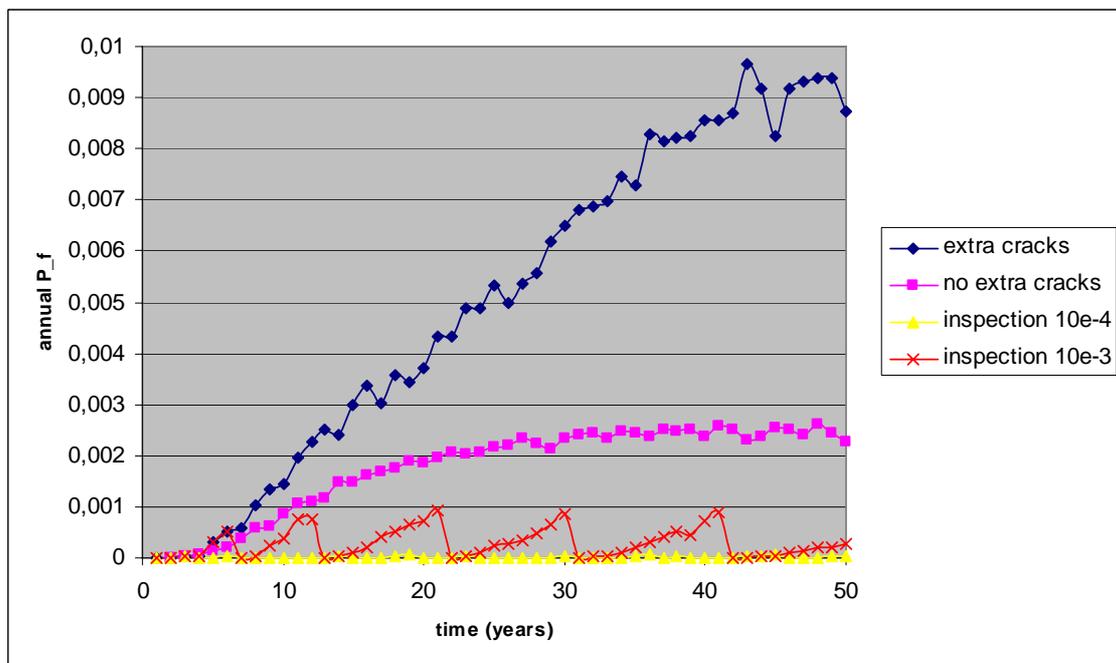


Figure 23. Annual probability of failure as function of time. Without extra crack initiation, with extra crack initiation - Linear [10 ; 25] and $\alpha_i = 3 \times 2/15$, and with inspections when $\Delta P_F^{\max} = 10^{-4}$ and $\Delta P_F^{\max} = 10^{-3}$.

Comment:

- The inspection time intervals are unchanged before the time where extra cracks initiate.
- The inspection time intervals become smaller when more cracks are initiated – but the effect of the inspections imply that when the extra inspections start early, then most of the critical ones are detected and therefore the inspection time intervals can again increase.
- A large effect is obtained using e.g. a linear model for extra crack initiation rate with extra cracks in the interval [10 ; 25] years. Here the increase in inspection time intervals becomes negligible in the time interval [20 ; 40] years (until the

effect of the extra cracks have disappeared). Figure 22 shows the annual initiation rate of crack initiations. The Weibull distributed crack initiation time, N_I and the extra linear crack initiation are clearly seen. Figure 23 shows the annual probability of failure as function of time without extra crack initiation, with extra crack initiation (linear [10 ; 25] and $\alpha_I = 3 \times 2/15$), and with inspections when $\Delta P_F^{\max} = 10^{-4}$ and $\Delta P_F^{\max} = 10^{-3}$. The annual probability of failure is seen to increase significantly when extra initiation of cracks is included. Using inspections it is seen that it is possible to obtain a maximum annual probability of failure below ΔP_F^{\max} .

Using model d) the fracture mechanical model including the extra initiation of cracks could be calibrated to the SN based approach. The parameters in the fracture mechanical model then become:

$$\mu_{I_0} = 3 \text{ years} \quad \text{and} \quad \mu_{\ln C_c} = -27.5$$

The reliability indices (based on accumulated probability of failure) are shown in figure 24. The resulting inspection plan with $\Delta P_F^{\max} = 10^{-4}$ and $\Delta P_F^{\max} = 10^{-3}$ are shown in tables 18 and 19.

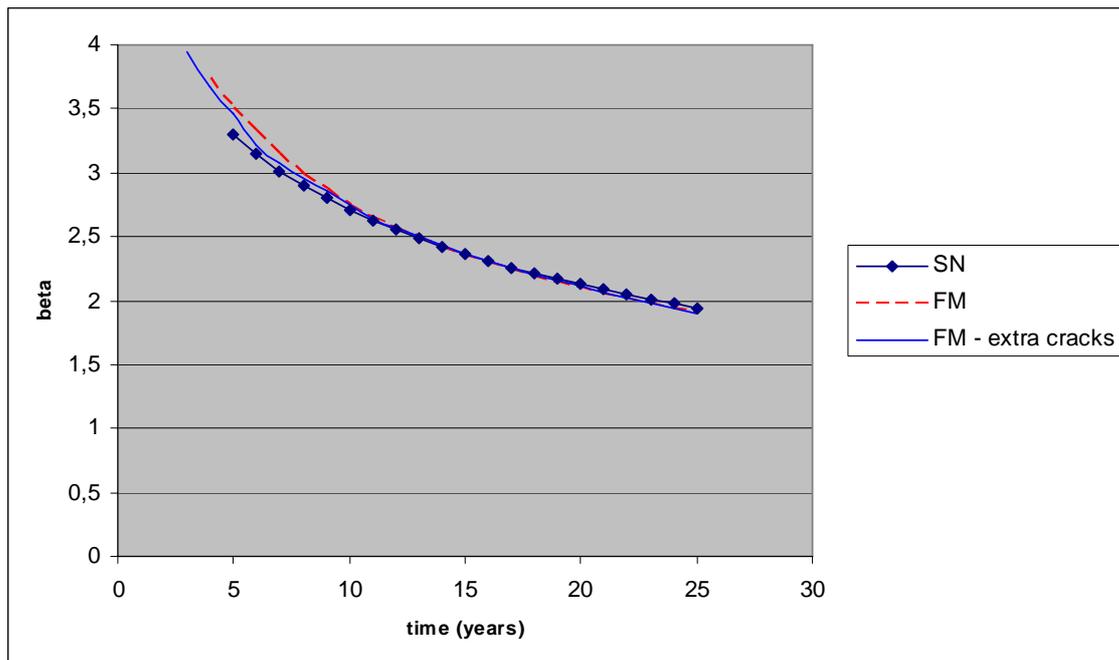


Figure 24. Reliability index (accumulated) as function of time for SN approach and calibrated FM-approach – without and with extra cracks included in calibration.

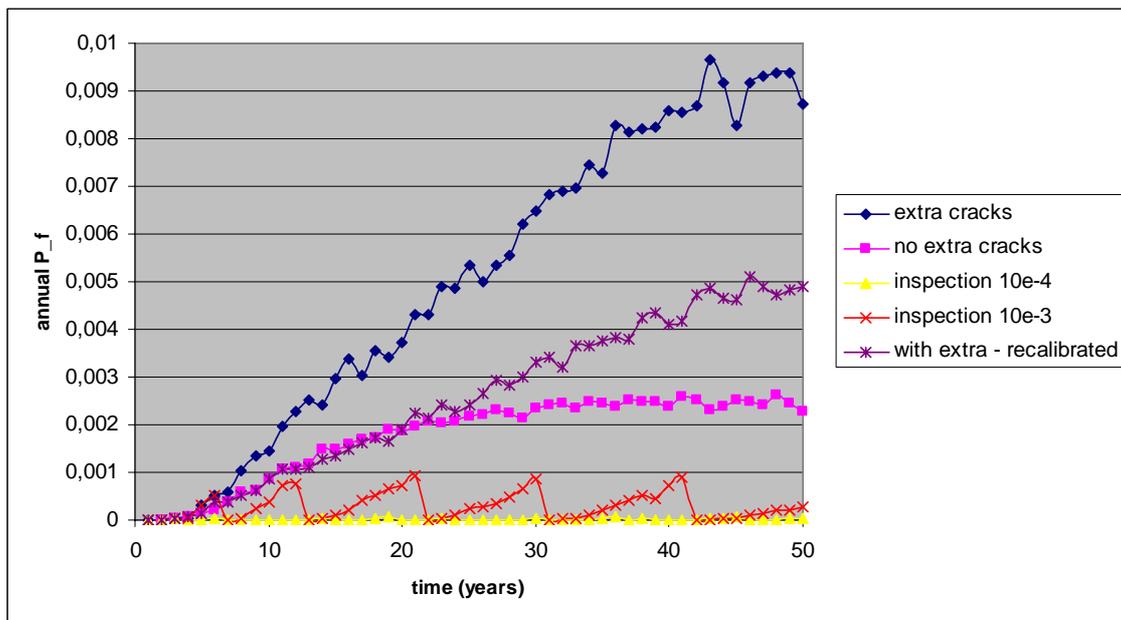


Figure 25. Annual probability of failure as function of time. As figure 7, but with re-calibrated model.

Model	α_I	Inspection times (upper values) and intervals (lower values) in years											
No extra cracks		4	6	9	12	17	23	32	41				
		4	2	3	3	5	6	9	9				
Linear [10 ; 25]	$3 \times 2/15$	4	5	7	9	11	15	20	24	28	32	39	46
		4	1	2	2	2	4	5	4	4	4	7	7
Linear [10 ; 25] re-calibrated	$3 \times 2/15$	4	5	8	11	16	22	27	34	43			
		4	1	3	3	5	6	5	7	9			

Table 19. Inspection times and time intervals – extra initiation of cracks – re-calibrated model used. $\Delta P_F^{\max} = 10^{-4}$.

Model	α_I	Inspection times (upper values) and intervals (lower values) in years											
No extra cracks		21											
		21											
Linear [10 ; 25]	$3 \times 2/15$	6	12	21	30	41							
		6	6	9	9	11							
Linear [10 ; 25] re-calibrated	$3 \times 2/15$	10	22	36									
		10	12	14									

Table 20. Inspection times and time intervals – extra initiation of cracks – re-calibrated model used. $\Delta P_F^{\max} = 10^{-3}$.

Comment:

- The inspection time intervals are larger with the re-calibrated model, but compared to the model without extra cracks, the inspection intervals have the wanted effect

Finally, an alternative model for the crack initiation time, N_I is used. N_I is assumed to be exponential distributed with expected value μ_{I_0} . If extra initiating cracks at the end of the lifetime is included, then a bath-tub like form of the total distribution of the crack initiation rate is obtained.

The fracture mechanical model including the extra initiation of cracks is calibrated to the SN based approach. The parameters in the fracture mechanical model then become:

$$\mu_{I_0} = 5 \text{ years} \quad \text{and} \quad \mu_{\ln C_c} = -27.5$$

The reliability indices (based on accumulated probability of failure) are shown in figure 264. The resulting inspection plan with $\Delta P_F^{\max} = 10^{-4}$ is shown in table 20.

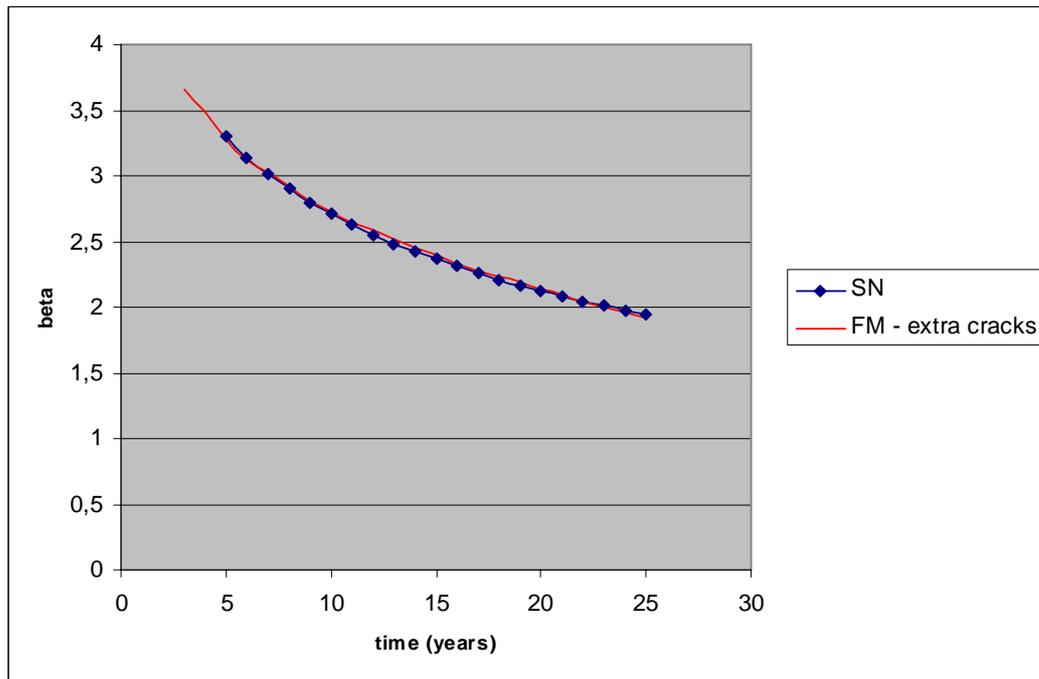


Figure 26. Reliability index (accumulated) as function of time for SN approach and calibrated FM-approach – without and with extra cracks included in calibration.

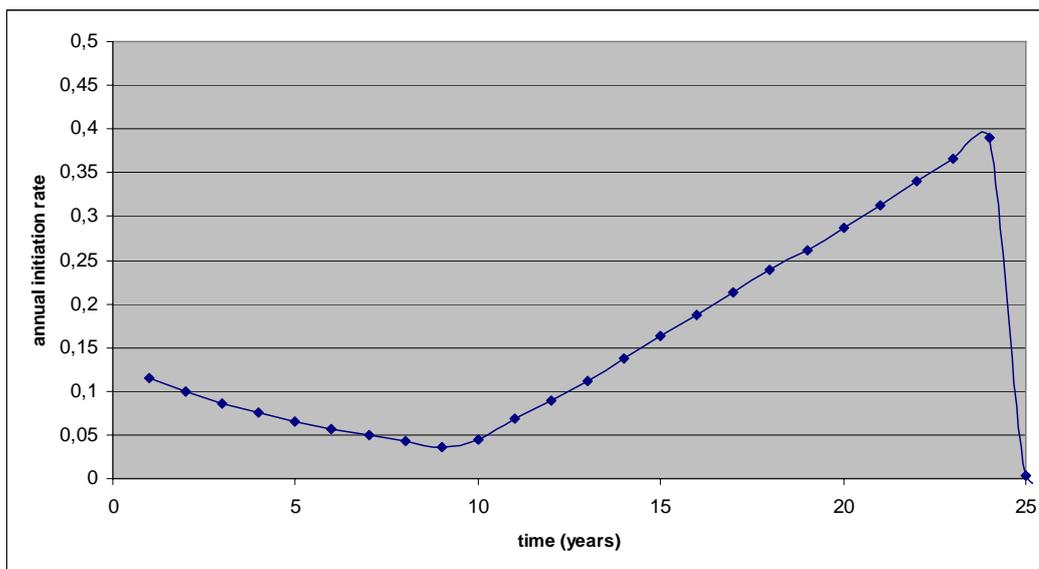


Figure 27. Annual rate of crack initiation – exponential distributed initial crack initiation and Linear model for extra crack initiation in [10 ; 25] and $\alpha_I = 3 \times 2/15$.

Model	α_I	Inspection times (upper values) and intervals (lower values) in years									
Linear [10 ; 25]	$3 \times 2/15$	3	5	8	1	18	23	30	37	47	
		3	2	3	4	6	5	7	7	10	

Table 20. Inspection times and time intervals – extra initiation of cracks. $\Delta P_F^{\max} = 10^{-4}$.

Comment:

- The inspection intervals have the ‘wanted’ effect

Summary / comparison of model a) – d)

- Only the model with extra initiation of cracks has the ‘wanted’ effect.
- The main reason that the inspection time intervals still increase in the other models a) - c) is the statistical effect of the inspection, namely that fast growing cracks are detected by the inspections – if not by the first inspection then by one of the following inspections.

5 Systems effects

In many situations there will be a number of fatigue crack critical details (components) in an offshore steel platform. In this section different models for these systems effects are discussed. The following aspects are considered:

- a. Assessment of the acceptable annual fatigue probability of failure for a particular component can be dependent on the number of fatigue critical components. The acceptable annual probability of fatigue failure of a component is obtained considering the importance of the component through the conditional probability of failure given failure of the component.
- b. Due to common loading, common model uncertainties and correlation between inspection qualities it can be expected that information obtained from inspection of one component can be used not only to update the inspection plan for that component, but also for other nearby components.
- c. In some cases the development of a crack in one component causes a stiffness reduction and an increased damping which imply that loads could be redistributed and thereby increase the stress ranges in some of the other fatigue critical details.

5.1 Aspect a – acceptable annual fatigue probability of failure

In order to assess the acceptable annual probability of fatigue failure for a component in a platform the probability of failure of the considered platform must be calculated conditional on fatigue failure of the considered joint. In section 2 the basic consideration for one component / critical detail is described. In this section systems effects are included.

The ‘deterministic’ importance of a fatigue failure is measured by the Residual Influence Factor, RIF defined by equation (1). The principal relation between RIF and annual collapse probability is illustrated in figure 2.

In section 2 it is also described how the individual joint acceptance criteria for the annual probability of joint fatigue failure can be determined as

$$\Delta P_F^{\max} = \frac{P_{AC}}{P_{COL|FAT_j}} \quad (22)$$

Such that the inspection plans must then satisfy

$$P_{FAT_j} \leq \Delta P_F^{\max} \quad (23)$$

for all years during the operational life of the platform.

A general relation between RSR and the probability of failure can be obtained considering e.g. the following general limit state function:

$$g(x) = R - bH^a \quad (24)$$

where R is the effective capacity of the platform, a is a shape factor typically equal to 2, b is an influence coefficient taking into account model uncertainty parameter and H^a is

a stochastic variable modeling the maximum annual value of the environmental load parameter.

The RSR value as evaluated by a push-over analysis can be related to characteristic values of R , a , b and H i.e. R_C , b_C and H_C in the following way

$$RSR = \frac{R_C}{b_C H_C^a} \quad (25)$$

Typically, it can be assumed that R and b can be modeled probabilistically as log-Normal distributed random variables and H^a as a Gumbel distributed random variable. The characteristic value for R , b and H^a could be defined as 5%, 50% and 99% quantile values of their probability distributions. The example relationship in figure 2 is obtained using $RSR = 1.8$.

In the considerations above only one fatigue critical component is considered. Often a number of components will be critical with respect to fatigue failure. In codes of practice usually requirements are only specified to check that individual fatigue critical components have a satisfactory safety. It is therefore not clear how to relate the code requirements to an acceptable system probability of failure for the whole structure considering more than one fatigue critical component. However, a first estimate can be obtained if it is assumed that N members are critical, the members contribute equally to the probability of failure and the system probability of failure is estimated by one of the following two possibilities:

- simple upper bound on the system probability of failure. Then

$$P_{AC,FAT} = \frac{1}{N} \frac{P_{AC}}{P_{COL|FAT}} \quad (26)$$

$P_{AC,FAT}$ is shown in figure 28 for $N=1, 2, 5$ and 10 critical components.

- approximate estimate of the system probability of failure. Then

$$P_{AC,FAT} = \frac{P_{AC}}{P_{SYS}} \quad (27)$$

where

$$P_{SYS} = 1 - \Phi_N(\beta_1, \beta_2, \dots, \beta_N; \rho) \quad (28)$$

with the reliability index for each member, β_i given fatigue failure is $\beta_i = -\Phi^{-1}(P_{COL|FAT_i})$ and the correlation coefficients in the correlation coefficient

matrix, ρ are obtained assuming that only the wave loading is common in different components. $P_{AC,FAT}$ estimated by (27) is shown in figure 29.

It is seen that the simple upper bounds in figure 28 for $N=1, 2, 5$ and 10 critical components give reasonable conservative estimates of the acceptable probability of fatigue failure.

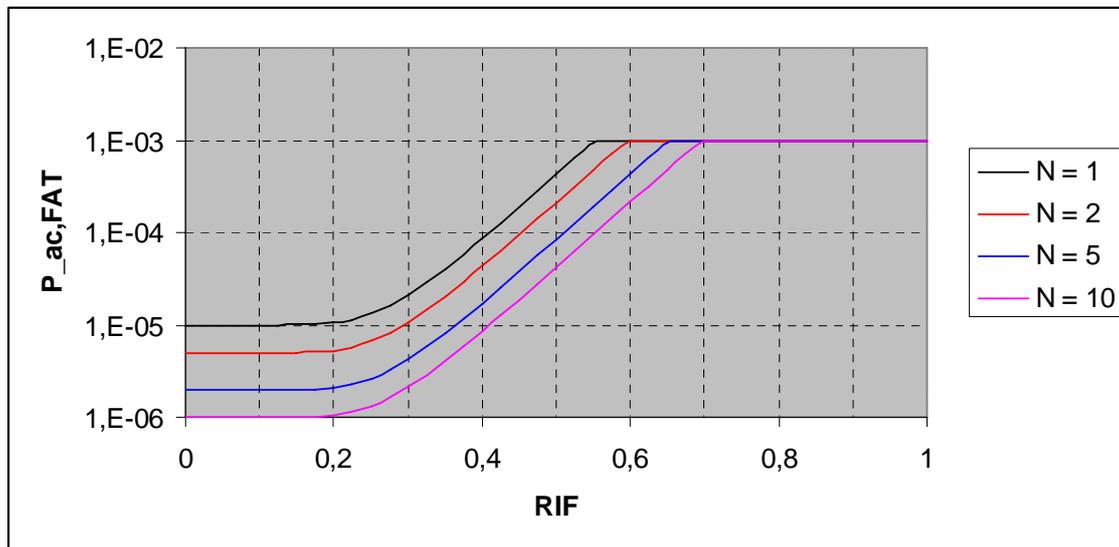


Figure 28. Maximum acceptable annual probability of fatigue failure, $P_{AC,FAT}$ as function of RIF (Residual Influence Factor) based on an upper bound on the probability of failure.

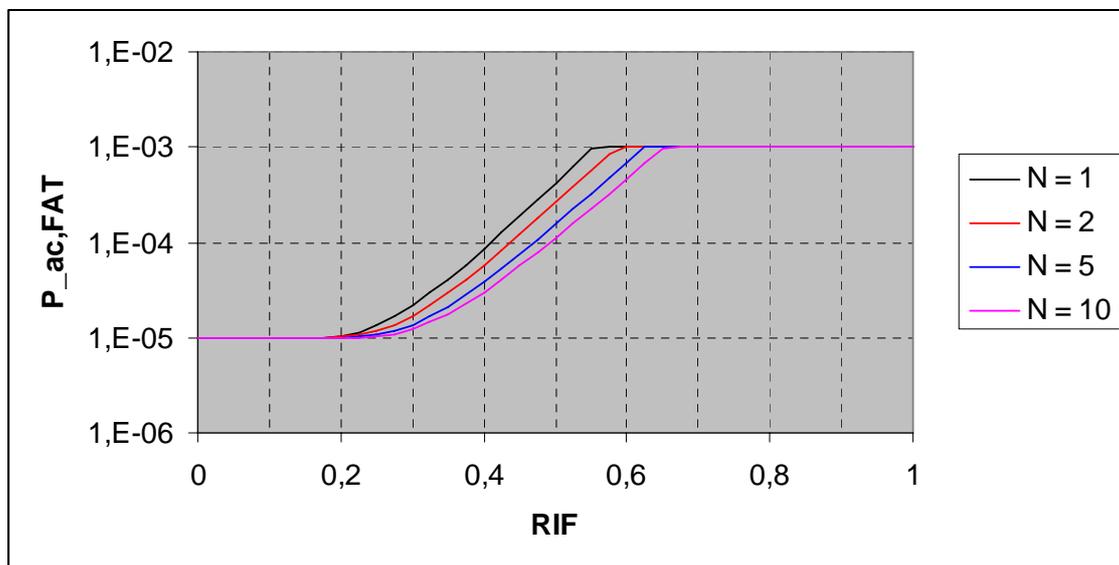


Figure 29. Maximum acceptable annual probability of fatigue failure, $P_{AC,FAT}$ as function of RIF (Residual Influence Factor) based on an approximate estimate of the probability of failure.

5.2 Aspect b – update inspection plan based on inspection of other components

Due to common loading, common model uncertainties and correlation between inspection qualities it can be expected that information obtained from inspections of one or more components can be used not only to update the inspection plan for these components, but also for other nearby components.

	Variable	Description	Distribution
Strength Variables	N_I	Number of stress cycles to initiation of crack	Weibull
	a_0	Initial crack length	Exponential
	$\ln C_C$	Crack growth parameter	Normal
	Y	Geometry function	LogNormal
Load Variables	X_{SCF}	Uncertainty stress range calculation	LogNormal
	X_{wave}	Uncertainty wave load	LogNormal
	a, b	Weibull parameter in long term stress range distribution	LogNormal
Inspection quality	c_d	Probability Of Detection curve POD – smallest detectable crack length	POD

Table 21. Stochastic variables for fracture mechanical analysis.

Table 21 shows the stochastic variables typically used in the fracture mechanical model. Considering as an example two fatigue critical components, the limit state functions corresponding to fatigue failure can be written:

$$g_1(\mathbf{X}_{Load,1}, \mathbf{X}_{Strength,1}, t) = a_{c,1} - a_1(\mathbf{X}_{Load,1}, \mathbf{X}_{Strength,1}, t) = 0 \quad (29)$$

$$g_2(\mathbf{X}_{Load,2}, \mathbf{X}_{Strength,2}, t) = a_{c,2} - a_2(\mathbf{X}_{Load,2}, \mathbf{X}_{Strength,2}, t) = 0 \quad (30)$$

where

$a_j(\mathbf{X}_{Load,j}, \mathbf{X}_{strength,j}, t)$ crack depth at time t for component j

$a_{c,j}$ critical crack depth for component j

$\mathbf{X}_{Load,j}$ load variables (X_{SCF} , X_{wave} , a and b) for component j

$\mathbf{X}_{strength,j}$ strength variables (N_I , a_0 , $\ln C_C$ and Y) for component j

The events corresponding to detection of a crack at time T can be written:

$$h_1(\mathbf{X}_{Load,1}, \mathbf{X}_{Strength,1}, c_{d,1}, T) = c_{d,1} - c_1(\mathbf{X}_{Load,1}, \mathbf{X}_{Strength,1}, T) \leq 0 \quad (31)$$

$$h_2(\mathbf{X}_{Load,2}, \mathbf{X}_{Strength,2}, c_{d,2}, T) = c_{d,2} - c_2(\mathbf{X}_{Load,2}, \mathbf{X}_{Strength,2}, T) \leq 0 \quad (32)$$

where

$c_j(\mathbf{X}_{Load,j}, \mathbf{X}_{strength,j}, c_{d,j}, T)$ crack length at time T for component j

$c_{d,j}$ smallest detectable crack length for component j

It is noted that the crack depth $a_j(t)$ and crack length $c_j(t)$ are related through the coupled differential equations in (15).

The stochastic variables in different components will typically be dependent as follows:

- The load related variables can be assumed fully dependent since the loading is common to most components. However, in special cases different types of components and components placed with a long distance between each other can be less dependent.
- The strength variables N_j , a_0 and $\ln C_C$ will typically be independent since the material properties are varying from component to component. However, some dependence can be expected for components fabricated with the same production techniques and from the same basic materials.
- The geometry function uncertainty modelled by Y will be fully dependent if the same type of fatigue critical details / components are considered and independent if two different types of fatigue critical details / components are considered.

Updated probabilities of failure of component 1 and 2 given no detection of cracks in detail 1 and 2 are

$$\begin{aligned}
 P_{F,1|1} &= \\
 &P(\text{failure of component 1 in time interval } [0, t] | \text{no detection in component 1 at time } T) = \\
 &P(g_1(\mathbf{X}_{Load,1}, \mathbf{X}_{Strength,1}, t) \leq 0 | h_1(\mathbf{X}_{Load,1}, \mathbf{X}_{Strength,1}, c_{d,1}, T) > 0)
 \end{aligned} \tag{33}$$

$$\begin{aligned}
 P_{F,2|2} &= \\
 &P(\text{failure of component 2 in time interval } [0, t] | \text{no detection in component 2 at time } T) = \\
 &P(g_2(\mathbf{X}_{Load,2}, \mathbf{X}_{Strength,2}, t) \leq 0 | h_2(\mathbf{X}_{Load,2}, \mathbf{X}_{Strength,2}, c_{d,2}, T) > 0)
 \end{aligned} \tag{34}$$

$$\begin{aligned}
 P_{F,2|1} &= \\
 &P(\text{failure of component 2 in time interval } [0, t] | \text{no detection in component 1 at time } T) = \\
 &P(g_2(\mathbf{X}_{Load,2}, \mathbf{X}_{Strength,2}, t) \leq 0 | h_1(\mathbf{X}_{Load,1}, \mathbf{X}_{Strength,1}, c_{d,1}, T) > 0)
 \end{aligned} \tag{35}$$

$$\begin{aligned}
 P_{F,1|2} &= \\
 &P(\text{failure of component 1 in time interval } [0, t] | \text{no detection in component 2 at time } T) = \\
 &P(g_1(\mathbf{X}_{Load,1}, \mathbf{X}_{Strength,1}, t) \leq 0 | h_2(\mathbf{X}_{Load,2}, \mathbf{X}_{Strength,2}, c_{d,2}, T) > 0)
 \end{aligned} \tag{36}$$

(33) and (34) represent situations where a component is updated with inspection of the same component. (35) and (36) represent situations where a component is updated with inspection of another component. The above formulas can easily be extended to cases where both components are inspected to where more components are inspected.

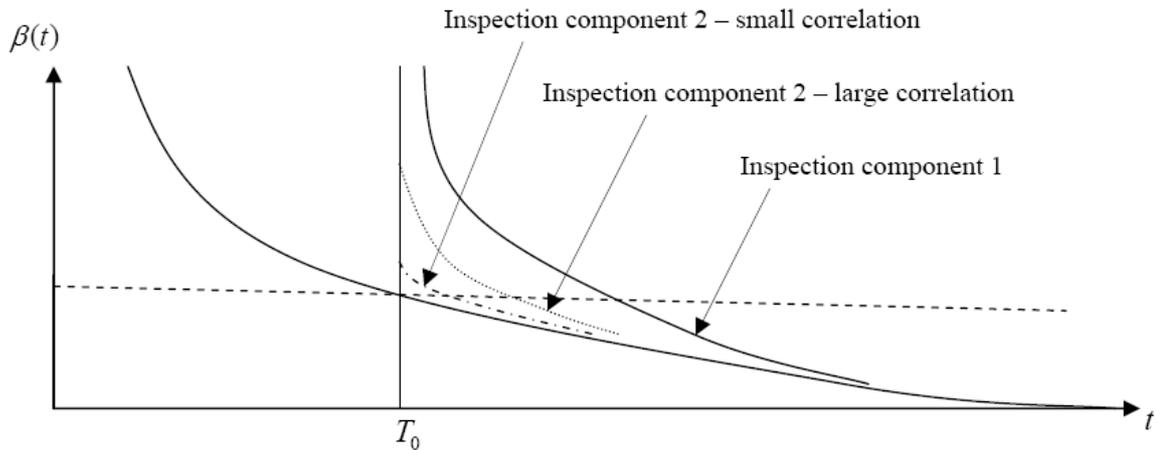


Figure 30. Reliability index as function of time for component no. 1 and updated reliability if inspection of component no. 1 at time T_0 , or of component no. 2 at time T_0 with large and small positive correlation with component no. 1.

The efficiency of updating the probability of fatigue failure for one component by inspection of another component depends on the degree of correlation between the stochastic variables as discussed above. Further, the relative importance of the load and the strength variables is important. If the load variables are highly uncertain and thus have high *COVs* then it can be expected that inspection of another components is efficient, because the highly correlated load variables accounts for a large part of the uncertainty in the failure events considered.

In figure 30 is illustrated the effect on inspection planning for a component if this component is inspected or if another nearby component is inspected. The largest effect on reliability updating and thus inspection planning is obtained inspecting the same component or inspection of another component with a large correlation with the considered component.

Thus, inspection of a few components can be expected to be of high value for all components if:

- The strength variables are correlated – and this can be the case if
 - the fatigue critical details / components are of the same type (e.g. cracks in tubular K-joints) and the components are placed geometrical close to each other,
 - the components are fabricated under similar conditions and with the same basic material.
- The load variables have a relatively high uncertainty compared to the strength variables, and the components are placed geometrical close to each other.

Considering a group of components the reliability-based inspection planning problem can now be generalised to

- choosing the components to be inspected
- determining the time intervals between inspections – time intervals are not necessary the same for all components
- choosing the inspection method(s) to be used (often the same inspection methods will be used for all inspections)

The generic inspection planning technique could be generalised such that inspection times are planned for all N components by including a few more generic parameters:

- number N of components which could be inspected
- correlation between all N components

A simplified generic inspection planning technique could be obtained if only inspection planning for one component at the time is made but using information from other inspected components. The following information is needed:

- number $N-1$ of other components with inspection information
- correlation the considered components and the other $N-1$ components
- inspection times for the other $N-1$ components (no detection of cracks are assumed)

The information on correlation between components could e.g. be given using the simplified scheme in table 22 where three levels of correlation are assumed.

Uncertainty type	Level 1	Level 2	Level 3
common load uncertainties (assuming the same level of COV_{wave} and COV_{SCF} in the considered components)	Yes	Yes	Yes
common strength model uncertainties related to Δ and Y	No	Yes	Yes
partly correlated material fatigue parameters $\ln C$ (e.g. correlation coefficient equal to 0.5)	No	No	Yes

Table 22. Levels of correlation between fatigue critical components.

5.3 Aspect c – effect of redistribution of load effects due to growing cracks

In some cases the development of a crack in one component causes a stiffness reduction which imply that loads are redistributed and thereby increased stress ranges in other fatigue critical details. This effect can be modelled in the limit state equation by introducing a multiplier $\alpha_1(a_2(t), a_3(t), \dots)$ on the stress ranges for component 1:

$$g_1(\mathbf{X}_{Load,1}, \mathbf{X}_{Strength,1}, \alpha_1(a_2(t), a_3(t), \dots), t) = 0 \quad (37)$$

As a simplification a multiplier corresponding to the redistribution when the crack depths in the relevant nearby details are equal to e.g. half the critical depth.

6 Summary

The basic principles in reliability and risk based inspection planning are described. The basic assumption made in risk / reliability based inspection planning is that a Bayesian approach can be used. The Bayesian approach and the no-crack detection assumption implies that the inspection time intervals usually become longer and longer. Further, inspection planning based on the RBI approach implies that single components are considered, one at the time, but with the acceptable reliability level assessed based on the consequence for the whole structure in case of fatigue failure of the component.

The following two aspects are considered with the aim to develop the risk based inspection approach for ageing structures, namely

- For aging platform several small cracks are often observed – implying an increased risk for crack initiation (and coalescence of small cracks) and increased crack growth. This should imply shorter inspection time intervals for ageing structures.
- Systems effects including
 - Assessment of the acceptable annual fatigue probability of failure for a particular component taking into account that there can be many fatigue critical components in a structure.
 - Due to common loading, common model uncertainties and correlation between inspection qualities it can be expected that information obtained from inspection of one component can be used not only to update the inspection plan for that component, but also for other nearby components.

Different approaches for updating inspection plans for older installations are proposed. The most promising method consists in increasing the rate of crack initiations at the end of the expected lifetime – corresponding to a bath-tub hazard rate effect. The approach is illustrated for welded steel details in platforms, and implies that inspection time intervals decrease at the end of the platform lifetime.

Data is needed to verify the increased crack initiation model. These data can be direct observations of cracks in older installations or indirect information from inspection programmes.

The different principal system effects are described, and a possible implantation in the generic inspection framework is described.

The approaches described is especially developed for inspection planning of fatigue cracks, but can also be used for various other deterioration processes where inspection is relevant, including corrosion, chloride ingress in concrete and possible corrosion of reinforcement and wear.

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